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Research Article

Improving the Efficiency of Ratio Estimators by Calibration Weightings

***Etebong P. Clement[1] and Elisha J. Inyang[2]**
[1,2]Department of Statistics, University of Uyo, P. M. B. 1017, Uyo - Nigeria
***Corresponding Author Email:** epclement@yahoo.com; [2]inyang.elisha@yahoo.com

It is observed that the performances of most improved ratio estimators depend on some optimality conditions that need to be satisfied to guarantee better estimator. This paper develops a new approach to ratio estimation that produces a more efficient class of ratio estimators that do not depend on any optimality conditions for optimum performance using calibration weightings. The relative performances of the proposed calibration ratio estimators are compared with a corresponding global [Generalized Regression (GREG)] estimator. Results of analysis showed that the proposed calibration ratio estimators are substantially superior to the traditional GREG-estimator with relatively small bias, mean square error, average length of confidence interval and coverage probability. In general, the proposed calibration ratio estimators are more efficient than all existing estimators considered in the study.

Keywords: efficiency comparison, existing estimators, global estimator, optimality, conditions, ratio estimator

INTRODUCTION

The ratio estimation has gained relevance in Statistical Estimation Theory than the regression estimation because of its improved precision in estimating the population or subpopulation parameters. But the regression estimator, in spite of its lesser practicability, seems to be holding a unique position due to its sound theoretical basis. The classical ratio and product estimators even though considered to be more useful in many practical situations in fields like Agriculture, Forestry, Economics and population studies have efficiencies which does not exceed that of the linear regression estimator.

This limitation has prompted most survey Statisticians to carry out several researches towards the modification of the existing ratio, product or classes of ratio and product estimators of the population mean in simple random sampling without replacement (SRSWOR) to provide better alternatives and improve efficiency. Among the authors who have proposed improved estimators include [Singh and Tailor (2003), Kadillar and Cingi (2006), Gupta and Shabbir (2008), Sharma and Tailor (2010), Diana *et al*. (2011), Solanki *et al*. (2012), Swain (2012), Choudhary and Singh (2012), Subramani and Kumarapandiyan (2012), Singh and Solanki (2012), Khare and Sinha (2012), Haq and Shabbir (2013), Shittu and Adepoju (2014)].

However, it has been observed that the performances of most of the proposed improved ratio estimators of the works cited above depend on some optimality conditions that need to be satisfied to guarantee better estimator. A unique approach to addressing these problems is by calibration estimation. The concept of calibration estimator was introduced by Deville and Sarndal (1992) in survey sampling. Calibration estimation is a method that uses auxiliary variable(s) to adjust the original design weights to improve the precision of survey estimates of population or subpopulation parameters. The calibration weights are chosen to minimize a given distance measure (or loss function) and these weights satisfy the constraints related auxiliary variable information. Calibration estimation has been studied by many survey Statisticians. A few key references are [Wu and Sitter (2001), Montanari and Ranalli (2005), Farrel and Singh (2005), Arnab and Singh (2005), Estavao and Sarndal (2006), Kott (2006), Kim *et al.* (2007), Sarndal (2007), Kim and Park (2010), Kim and Rao (2012), Rao *et al.* (2012), Koyuncu and Kadilar (2013), Clement *et al.* (2014), Clement and Enang (2015a,b, 2017), Clement (2016, 2017a,b,c, 2018a,b, 2020, 2021), Enang and Clement (2020)].

This paper develops the theory of calibration estimators for ratio estimation and proposes a class of calibration ratio-type estimators based on Subramani and Kumarapandiyan (2012) ratio-type estimators under the simple random sampling without replacement (SRSWOR).

METHODOLOGY

Subramani and Kumarapandiyan (2012) proposed a set of four ratio-type estimators using known values of the population quartiles of the auxiliary variable and their functions as given by the following:

(i) $\hat{\bar{Y}}_1 = \bar{y}\left[\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right]$ (1)

$$B(\hat{\bar{Y}}_1) = \gamma\bar{Y}\left(\vartheta_1^2 C_x^2 - \vartheta_1 C_x C_y \rho\right)$$
$$MSE(\hat{\bar{Y}}_1) = \gamma\bar{Y}^2\left(C_y^2 + \vartheta_1^2 C_x^2 - 2\vartheta_1 C_x C_y \rho\right)$$

(ii) $\hat{\bar{Y}}_2 = \bar{y}\left[\frac{\bar{X}+Q_\alpha}{\bar{x}+Q_\alpha}\right]$ (2)

$$B(\hat{\bar{Y}}_2) = \gamma\bar{Y}\left(\vartheta_2^2 C_x^2 - \vartheta_2 C_x C_y \rho\right)$$
$$MSE(\hat{\bar{Y}}_1) = \gamma\bar{Y}^2\left(C_y^2 + \vartheta_2^2 C_x^2 - 2\vartheta_2 C_x C_y \rho\right)$$

(iii) $\hat{\bar{Y}}_3 = \bar{y}\left[\frac{\bar{X}+Q_\eta}{\bar{x}+Q_\eta}\right]$ (3)

$$B(\hat{\bar{Y}}_3) = \gamma\bar{Y}\left(\vartheta_3^2 C_x^2 - \vartheta_3 C_x C_y \rho\right)$$
$$MSE(\hat{\bar{Y}}_3) = \gamma\bar{Y}^2\left(C_y^2 + \vartheta_3^2 C_x^2 - 2\vartheta_3 C_x C_y \rho\right)$$

(iv) $\hat{\bar{Y}}_4 = \bar{y}\left[\frac{\bar{X}+Q_\varphi}{\bar{x}+Q_\varphi}\right]$ (4)

$$B(\hat{\bar{Y}}_4) = \gamma\bar{Y}\left(\vartheta_4^2 C_x^2 - \vartheta_4 C_x C_y \rho\right)$$
$$MSE(\hat{\bar{Y}}_4) = \gamma\bar{Y}^2\left(C_y^2 + \vartheta_4^2 C_x^2 - 2\vartheta_4 C_x C_y \rho\right)$$

where Q_1 is the first quartile or the lower quartile, Q_3 is the third quartile or the upper quartile, $Q_\alpha = (Q_3 - Q_1)$ denotes the inter-quartile range, $Q_\eta = (Q_3 - Q_1)/2$ denotes the semi-quartile range, $Q_\varphi = (Q_3 + Q_1)/2$ denotes the quartile average, $\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\vartheta_1 = \left(\frac{\bar{X}}{\bar{X}+Q_3}\right)$, $\vartheta_2 = \left(\frac{\bar{X}}{\bar{X}+Q_\alpha}\right)$, $\vartheta_3 = \left(\frac{\bar{X}}{\bar{X}+Q_\eta}\right)$, and $\vartheta_4 = \left(\frac{\bar{X}}{\bar{X}+Q_\varphi}\right)$.

These proposed ratio estimators by Subramani and Kumarapandiyan (2012), [as given in equations (1) – (4) above], were more efficient than the Sisodia and Dwivedi (1981), Singh and Tailor (2003), Singh *et al.* (2004) and Yan and Tian (2010) ratio-type estimators, except the linear regression estimator.

The calibration estimator of population mean under the simple random sampling without replacement (SRSWOR) according to Deville and Sarndal (1992), is given by

$$\hat{\bar{Y}}_{DS} = \sum_{i=1}^{n} W_i \bar{y} \quad (5)$$

where W_i are the calibration weights

Motivated by Deville and Sarndal (1992) and Subramani and Kumarapandiyan (2012), a new set of ratio estimators of population mean using known values of the population quartiles of the auxiliary variable and their functions is proposed under the calibration estimation as given by the following:

(i) $\hat{\bar{Y}}_\xi^* = \sum_{i=1}^{n} W_i^* \bar{y}\left[\frac{\bar{X}+Q_3}{\bar{x}+Q_3}\right]$ (6)

(ii) $\hat{\bar{Y}}_\alpha^* = \sum_{i=1}^{n} W_i^* \bar{y}\left[\frac{\bar{X}+Q_\alpha}{\bar{x}+Q_\alpha}\right]$ (7)

(iii) $\hat{\bar{Y}}_\eta^* = \sum_{i=1}^{n} W_i^* \bar{y}\left[\frac{\bar{X}+Q_\eta}{\bar{x}+Q_\eta}\right]$ (8)

(iv) $\hat{\bar{Y}}_\varphi^* = \sum_{i=1}^{n} W_i^* \bar{y}\left[\frac{\bar{X}+Q_\varphi}{\bar{x}+Q_\varphi}\right]$ (9)

where W_i^* is the calibration weights such that $0 < W_i^* \leq 1$.

The calibration weights W_i^* are chosen such that a chi-square type loss functions of the form:

$$L = \sum_{i=1}^{n} \frac{(W_i^* - d_i)^2}{d_i q_i} \quad (10)$$

is minimized while satisfying the calibration constraint

$$\sum_{i=1}^{n} W_i^* \bar{X} = \beta_1(x) \quad (11)$$

where q_i is the tuning parameter, $\beta_1(x)$ is the coefficient of skewness of the auxiliary variable and d_i is the design weight denoted by $d_i = 1/\pi_i$ where π_i is the inclusion probability denoted by $\pi_i = n/N$ so that $d_i = N/n$.
Minimizing the loss functions (10) subject to the calibration constraint (11) gives the calibration weights in simple random sampling as given by:

$$W_i^* = d_i + \frac{q_i d_i \bar{X}}{\sum_{i=1}^{n} q_i d_i \bar{X}^2}\left(\beta_1(x) - \sum_{i=1}^{n} d_i \bar{X}\right) \quad (12)$$

Substituting (12) into equations (6), (7), (8), and (9) respectively and setting $q_i = \bar{X}^{-1}$ gives the proposed calibration ratio estimators under simple random sampling without replacement (SRSWOR) as follows:

$$(i) \quad \hat{\bar{Y}}_\xi^* = \frac{\sum_{i=1}^{n} d_i \bar{y}}{\sum_{i=1}^{n} d_i \bar{X}}\left[\frac{\bar{X} + Q_3}{\bar{x} + Q_3}\right]\beta_1(x) \quad (13)$$

$$(ii) \quad \hat{\bar{Y}}_\alpha^* = \frac{\sum_{i=1}^{n} d_i \bar{y}}{\sum_{i=1}^{n} d_i \bar{X}}\left[\frac{\bar{X} + Q_\alpha}{\bar{x} + Q_\alpha}\right]\beta_1(x) \quad (14)$$

$$(iii) \quad \hat{\bar{Y}}_\eta^* = \frac{\sum_{i=1}^{n} d_i \bar{y}}{\sum_{i=1}^{n} d_i \bar{X}}\left[\frac{\bar{X} + Q_\eta}{\bar{x} + Q_\eta}\right]\beta_1(x) \quad (15)$$

$$(iv) \quad \hat{\bar{Y}}_\varphi^* = \frac{\sum_{i=1}^{n} d_i \bar{y}}{\sum_{i=1}^{n} d_i \bar{X}}\left[\frac{\bar{X} + Q_\varphi}{\bar{x} + Q_\varphi}\right]\beta_1(x) \quad (16)$$

To the first order degree of approximation; the biases and the mean square errors (MSEs) of the proposed set of estimators are given respectively as follows:

(i)
$$B(\hat{\bar{Y}}_\xi^*) = \gamma \bar{Y} \omega\left(\vartheta_\xi^2 C_x^2 - \vartheta_\xi C_x C_y \rho\right)$$
$$MSE\left(\hat{\bar{Y}}_\xi^*\right) = \gamma \bar{Y}^2 \omega^2\left(C_y^2 + \vartheta_\xi^2 C_x^2 - 2\vartheta_\xi C_x C_y \rho\right) \quad (17)$$

(ii)
$$B(\hat{\bar{Y}}_\alpha^*) = \gamma \bar{Y} \omega\left(\vartheta_\alpha^2 C_x^2 - \vartheta_\alpha C_x C_y \rho\right)$$
$$MSE\left(\hat{\bar{Y}}_\alpha^*\right) = \gamma \bar{Y}^2 \omega^2\left(C_y^2 + \vartheta_\alpha^2 C_x^2 - 2\vartheta_\alpha C_x C_y \rho\right) \quad (18)$$

(iii)
$$B(\hat{\bar{Y}}_\eta^*) = \gamma \bar{Y} \omega\left(\vartheta_\eta^2 C_x^2 - \vartheta_\eta C_x C_y \rho\right)$$
$$MSE\left(\hat{\bar{Y}}_\eta^*\right) = \gamma \bar{Y}^2 \omega^2\left(C_y^2 + \vartheta_\eta^2 C_x^2 - 2\vartheta_\eta C_x C_y \rho\right) \quad (19)$$

(iv)
$$B(\hat{\bar{Y}}_\varphi^*) = \gamma \bar{Y} \omega\left(\vartheta_\varphi^2 C_x^2 - \vartheta_\varphi C_x C_y \rho\right)$$
$$MSE\left(\hat{\bar{Y}}_\varphi^*\right) = \gamma \bar{Y}^2 \omega^2\left(C_y^2 + \vartheta_\varphi^2 C_x^2 - 2\vartheta_\varphi C_x C_y \rho\right) \quad (20)$$

where $\gamma = \left(\frac{1}{n} - \frac{1}{N}\right)$, $\omega = \left(\frac{\beta_1(x)}{\bar{X}}\right)$, $\vartheta_\xi = \left(\frac{\bar{X}}{\bar{X}+Q_3}\right)$, $\vartheta_\alpha = \left(\frac{\bar{X}}{\bar{X}+Q_\alpha}\right)$, $\vartheta_\eta = \left(\frac{\bar{X}}{\bar{X}+Q_\eta}\right)$, and $\vartheta_\varphi = \left(\frac{\bar{X}}{\bar{X}+Q_\varphi}\right)$

ANALYTICAL STUDY

Efficiency comparison

In this section, the MSEs of some existing estimators are compared with the MSEs of the new estimators. For clarity and convenience of the targeted readership, let the biases and MSEs of the set of new estimators discussed in section 2 be represented in a single class as follows:

$$B\left(\hat{\bar{Y}}_i^*\right) = \gamma \bar{Y} \omega\left(\vartheta_i^2 C_x^2 - \vartheta_i C_x C_y \rho\right)$$
$$MSE\left(\hat{\bar{Y}}_i^*\right) = \gamma \bar{Y}^2 \omega^2\left(C_y^2 + \vartheta_i^2 C_x^2 - 2\vartheta_i C_x C_y \rho\right) \quad (21)$$

where $i = 1,2,3,4$ and

$$\left.\begin{aligned} \vartheta_1 &= \left(\frac{\bar{X}}{\bar{X} + Q_3}\right) \\ \vartheta_2 &= \left(\frac{\bar{X}}{\bar{X} + Q_\alpha}\right) \\ \vartheta_3 &= \left(\frac{\bar{X}}{\bar{X} + Q_\eta}\right) \\ \vartheta_4 &= \left(\frac{\bar{X}}{\bar{X} + Q_\varphi}\right) \end{aligned}\right\} \quad (22)$$

Similarly, let the biases and MSEs of the set of modified ratio estimators proposed by Subramani and Kumarapandiyan (2012) be represented in a single class as follows:

$B(\hat{\bar{Y}}_i^*) = \gamma\bar{Y}\left(\vartheta_i^2 C_x^2 - \vartheta_i C_x C_y \rho\right)$

$$MSE\left(\hat{\bar{Y}}_i^*\right) = \gamma\bar{Y}^2\left(C_y^2 + \vartheta_i^2 C_x^2 - 2\vartheta_i C_x C_y \rho\right) \quad (23)$$

where ϑ_i is as defined in equation (22).

Subramani and Kumarapandiyan (2012) estimators

It is observed from equations (21) and (23), that the new calibration ratio estimators would be more efficient than the existing modified ratio estimators of Subramani and Kumarapandiyan (2012) if

$$\omega^2 \leq 1 \quad (24)$$

Classical ratio estimator

The classical ratio estimator of mean by Hansen *et al.* (1946) is given by

$$\hat{\bar{Y}}_R = \frac{\bar{y}}{\bar{x}}\bar{X} \quad (25)$$

The bias and MSE are respectively given by

$B(\hat{\bar{Y}}_R) = \gamma\bar{Y}\left(C_x^2 - C_x C_y \rho\right)$ and

$$MSE\left(\hat{\bar{Y}}_R\right) = \gamma\bar{Y}^2\left(C_y^2 + C_x^2 - 2C_x C_y \rho\right) \quad (26)$$

It is observed from equations (21) and (26), that the new calibration ratio estimators would be more efficient than the classical ratio estimator of Hansen *et al.* (1946) if

$$\omega^2 \leq \frac{(\phi^2 - 2\phi\rho + 1)}{(\vartheta_i^2\phi^2 - 2\vartheta_i\phi\rho + 1)} \quad (27)$$

where $\phi = \left(C_x/C_y\right)$

Classical regression estimator

The classical regression estimator proposed by Hansen *et al.* (1953) is given by

$$\hat{\bar{Y}}_{lr} = \bar{y} + b(\bar{X} - \bar{x}) \quad (28)$$

where $b = \left(\frac{S_{xy}}{S_x^2}\right)$ is the sample regression coefficient of y on x with

$$MSE\left(\hat{\bar{Y}}_{lr}\right) = \gamma\bar{Y}^2 C_y^2(1 - \rho^2) \quad (29)$$

It is observed from equations (21) and (29), that the new calibration ratio estimators would be more efficient than the classical regression estimator of Hansen *et al.* (1953) if

$$\omega^2 \leq \frac{(1 - \rho^2)}{(\vartheta_i^2\phi^2 - 2\vartheta_i\phi\rho + 1)} \quad (30)$$

where $\phi = \left(C_x/C_y\right)$

The percent relative efficiency (PRE)

In this section, the percent relative efficiency (PRE) of members of the proposed class of calibration ratio estimators and the existing estimators considered in this study are computed and presented in Table 2. The percent relative efficiency (PRE) of an estimator θ with respect to the classical ratio estimator ($\hat{\bar{Y}}_R$) is defined by

$$PRE\left(\theta, \hat{\bar{Y}}_R\right) = \frac{Var\left(\hat{\bar{Y}}_R\right)}{Var(\theta)} \times 100 \quad (31)$$

EMPIRICAL STUDY

To investigate the theoretical results, as well as, test the optimality and efficiency performance of the new calibration estimators over other existing estimators considered in this study, the data of the following populations [as adapted from Subramani and Kumarapandiyan (2012)] were used.

Population I

X is Fixed Capital and Y is Output for 80 factories in a region

$N = 80$ $n = 20$ $\bar{Y} = 51.8264$ $\bar{X} = 11.2646$ $\rho = 0.9413$

$C_x = 0.7507$ $C_y = 0.3542$ $S_x = 8.4563$ $S_y = 18.3569$ $\beta_{1(x)} = 1.0500$

$\beta_{2(x)} = 0.0634$ $Q_3 = 16.975$ $Q_\alpha = 16.975$ $Q_\Omega = 5.9125$ $Q_\varphi = 11.0625$

[Murthy (1967)]

Population II

X is data on number of workers and Y is Output for 80 factories in a region.

$N = 80$ $n = 20$ $\bar{Y} = 51.8264$ $\bar{X} = 2.8513$ $\rho = 0.9150$

$C_x = 0.9484$ $C_y = 0.3542$ $S_x = 2.7042$ $S_y = 18.3569$ $\beta_{1(x)} = 0.6978$

$\beta_{2(x)} = 1.3005$ $Q_3 = 4.4750$ $Q_\alpha = 3.6150$ $Q_\Omega = 1.8075$ $Q_\varphi = 2.6675$

[Murthy (1967)]

Table 1: Bias and MSE of the proposed estimators

Estimators	Population I		Population II	
	Bias	MSE	Bias	MSE
$\hat{\bar{Y}}_\xi^*$	0.0018	0.0135	0.0079	0.1354
$\hat{\bar{Y}}_\alpha^*$	0.0022	0.0135	0.0187	0.1766
$\hat{\bar{Y}}_\eta^*$	0.0142	0.0347	0.0708	0.5197
$\hat{\bar{Y}}_\varphi^*$	0.0031	0.0143	0.0386	0.2892

Table 2: Performance of estimators from analytical study

Estimators		Population I		Population II	
		MSE	PRE	MSE	PRE
Classical ratio		18.9764	100	41.3170	100
Classical Regression		1.4400	1,317.8056	2.0572	2,008.4095
Subramani & Kumarapandiyan (**2012**)	$\hat{\bar{Y}}_1$	1.5562	1,219.4062	2.2604	1,827.8623
	$\hat{\bar{Y}}_2$	1.5486	1,225.3909	2.9489	1,401.0987
	$\hat{\bar{Y}}_3$	3.9830	476.4348	8.6761	476.2163
	$\hat{\bar{Y}}_4$	1.6470	1,152.1797	4.8281	855.7611
Proposed	$\hat{\bar{Y}}_\xi^*$	0.0135	140,565.9259	0.1354	30,514.7711
	$\hat{\bar{Y}}_\alpha^*$	0.0135	140,565.9259	0.1766	23,395.8097
	$\hat{\bar{Y}}_\eta^*$	0.0347	54,687.0317	0.5197	7,950.1636
	$\hat{\bar{Y}}_\varphi^*$	0.0143	132,702.0979	0.2892	14,286.6528

SIMULATION STUDY

Background and analytical set-up

The data used is obtained from the 2005 socio-economic household survey of Akwa Ibom State conducted by the ministry of economic development, Uyo, Akwa Ibom State, Nigeria [Akwa Ibom State Government (2005)]. The study variable Y, represents the household expenditure on food and auxiliary variable X, represents the household income.

An assisting model of the form: $y_i = \beta_0 + \beta_1 x_i + e_i$ was designed for the calibration estimators, where e are independently generated by the standard normal distribution.

The simulation study was conducted using the R-statistical package. There were $B = 1{,}500$ for the b-th run $(b = 1, 2, \ldots, B)$, a Bernoulli sample was drawn where each unit was selected into the sample independently, with inclusion probability $\pi_i = n/N$. For simplicity the tuning parameter q_i was set to unity $(q_i = 1)$ and $n = 300$. The corresponding Greg-estimator and calibration ratio estimators of $\bar{Y}$ are computed: $\bar{Y}_{GREG}^{*(b)}$, $\hat{\bar{Y}}_\xi^{*(b)}$, $\hat{\bar{Y}}_\alpha^{*(b)}$, $\hat{\bar{Y}}_\eta^{*(b)}$ and $\hat{\bar{Y}}_\varphi^{*(b)}$. The results of the analysis are given in table 3.

Comparisons with a Global Estimator

The concept of calibration estimators proposed by Deville and Sarndal (1992) is simply a class of linearly weighted estimators, of which the Generalized Regression (GREG) estimator is a special member. Deville and Sarndal (1992) have shown that all calibration estimators are asymptotically equivalent to the GREG-estimator.
In this section, the efficiency performance of the new calibration ratio estimators is compared with a corresponding global estimator [Generalized Regression (GREG) estimator] and the results of analysis are presented in Table 3.

Simulation evaluation

Let $\hat{\hat{Y}}_i^{*(b)}$ be the estimate of $\hat{\hat{Y}}_i^*$ in the *b-th* simulation run; $b = 1,2,\ldots,B(= 1{,}500)$, for a given estimator (say) $\hat{\hat{Y}}_i^*$. To compare the efficiency performance of the new calibration ratio estimators with the GREG-estimator, the following criteria; bias (*B*), mean square error (*MSE*), average length of confidence interval (*AL*) and coverage probability (*CP*) of $\hat{\hat{Y}}_i^*$ were used. Each measure is calculated as follows:

(i) $B\left(\hat{\hat{Y}}_i^*\right) = \overline{\hat{\hat{Y}}_i^*} - \hat{\hat{Y}}_i^{*(b)}$

where $\overline{\hat{\hat{Y}}_i^*} = \frac{1}{B}\sum_{b=1}^{B}\hat{\hat{Y}}_i^{*(b)}$

(ii) $MSE\left(\hat{\hat{Y}}_i^*\right) = \sum_{b=1}^{B}\left(\hat{\hat{Y}}_i^{*(b)} - \overline{\hat{\hat{Y}}_i^*}\right)^2 / B$

where $\hat{\hat{Y}}_i^{*(b)}$ is the estimated total based on sample b and $B$is the total number of samples drawn for the simulation.

(iii) $CP\left(\hat{\hat{Y}}_i^*\right) = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{\hat{Y}}_L^{*(b)} < \hat{\hat{Y}}_i^{*(b)} < \hat{\hat{Y}}_U^{*(b)}\right)$

where $\hat{\hat{Y}}_L^{*(b)}$ is the lower confidence limit and $\hat{\hat{Y}}_U^{*(b)}$ is the upper confidence limit.
A coverage probability of 95% confidence interval is the ratio of the number of times the true population total is included in the interval to the total number of runs or the number of replicates. For each estimator of $\hat{\hat{Y}}_i^*$, a 95% confidence interval $\left(\hat{\hat{Y}}_L^{*(b)}, \hat{\hat{Y}}_U^{*(b)}\right)$ is constructed, where

$$\hat{\hat{Y}}_L^{*(b)} = \hat{\hat{Y}}_i^{*(b)} - 1.96\sqrt{V\left(\hat{\hat{Y}}_i^{*(b)}\right)},$$

$$\hat{\hat{Y}}_U^{*(b)} = \hat{\hat{Y}}_i^{*(b)} + 1.96\sqrt{V\left(\hat{\hat{Y}}_i^{*(b)}\right)} \text{ and}$$

$$V\left(\hat{\hat{Y}}_i^{*(b)}\right) = \frac{1}{B-1}\sum_{m=1}^{B}\left(\hat{\hat{Y}}_i^{*(b)} - \overline{\hat{\hat{Y}}_i^*}\right)^2$$

(iv) $AL\left(\hat{\hat{Y}}_i^*\right) = \frac{1}{B}\sum_{b=1}^{B}\left(\hat{\hat{Y}}_U^{*(b)} - \hat{\hat{Y}}_L^{*(b)}\right)$

Table 3: Performance of estimators from simulation study

Estimators		*B*	*MSE*	*AL*	*CP*
GREG		0.0082	84226	902.25	0.9174
Proposed	$\hat{\hat{Y}}_\xi^*$	0.0062	41182	762.46	0.9076
	$\hat{\hat{Y}}_\alpha^*$	0.0065	41696	770.06	0.9085
	$\hat{\hat{Y}}_\eta^*$	0.0073	42012	779.98	0.9102
	$\hat{\hat{Y}}_\varphi^*$	0.0068	41904	774.92	0.9094

DISCUSSION OF RESULTS

The results of analytical study presented in Table 2 showed that all the proposed calibration ratio estimators are more efficient than both the classical regression estimator and all modified ratio-type estimators proposed by Subramani and Kumarapandiyan (2012) under the two populations considered in the study with relatively high percent gains in efficiency. Again, it is observed that all the proposed calibration ratio estimators are almost unbiased as is reflected by their respective bias in Table 1 under the two population considered in the study.

For the simulation evaluation, the results of the residual diagnostics showed the R^2 value as 0.7489 indicating that the model is significant. The correlation between the study variable Y and the auxiliary variable X is $\rho_{xy} = 0.8934$ is strong and sufficient implying that the calibration ratio estimators would provide better estimates of the population mean.

Analysis for the comparison of performance of estimators showed that the biases of 0.62 percent, 0.65 percent, 0.73 percent, 0.68 percent, and 0.82 percent respectively for the four calibration ratio estimators and the GREG-estimator are negligible. But the bias of the GREG-estimator though negligible is the most biased among the estimators while the bias of the proposed calibration ratio estimator number one is the least biased among all the estimators considered. The variance for the GREG-estimator is significantly larger than those of the four calibration ratio estimators, as is indicated by their respective mean square errors in table 3. The average length of the confidence interval for each of the calibration ratio estimators is significantly smaller than that of the GREG-estimator. Similarly, the coverage probability of each of the calibration ratio estimators is also smaller than that of the GREG-estimator. These results showed that there is greater variation in the estimates made by the GREG-estimator than the calibration ratio estimators.

Within the calibration ratio estimators, calibration ratio estimator number one possesses better appealing statistical properties than the other three calibration ratio estimators as is reflected in table 3.

In general, the proposed calibration ratio estimators are substantially superior to the traditional GREG-estimator with relatively small bias, mean square error, average length of confidence interval and coverage probability.

CONCLUSION

This paper develops the concept of calibration estimator for ratio estimation and proposes a set of four calibration ratio estimators of population mean under the simple random sampling without replacement (SRSWOR) based on Subramani and Kumarapandiyan (2012) ratio-type estimators. Their MSEs are derived under large sample approximation and compared with those of related existing estimators in theory. Analytical and numerical results showed that the four proposed calibration ratio estimators are each always more efficient than the classical regression estimator and all related existing estimators considered in the study.

A comparison with a corresponding global estimator [the Generalized Regression (GREG) estimator] showed that the four proposed calibration ratio estimators are each substantially superior to the traditional GREG-estimator with relatively small bias, mean square error, average length of confidence interval and coverage probability.

These results proved the dominance of the new proposal over existing estimators and thus provide better alternative estimators in practical situations.

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