International Journal of Statistics and Mathematics
Vol. 4(1), pp. 099-107, August, 2017. © www.premierpublishers.org. ISSN: 2375-0499

IJSM

Research Article

Exploring Queuing Theory to Minimize Traffic Congestion Problem in Calabar-Highway by IBB Road, Cross River State

[*1]Akra, Ukeme Paulinus, [2]Ntekim, Offiong Edet,

1. Department of Statistics, University of Calabar, Calabar, Nigeria
2. Department of Mathematics, University of Calabar, Calabar, Nigeria

*__Corresponding author__ email: ukemeakra@gmail.com

Traffic congestion has been a serious problem that drivers are facing especially in Calabar – highway by IBB road intersection. In this paper, emphasis is placed on model formation and derivation of some parameters that will help to facilitate the flow of vehicles in this intersection to reduce traffic congestion. The channel considered in this research is multiple queue single servers. We derived variance waiting time of vehicles in the queue and in the system, expected number of vehicles in the queue and in the system waiting for service, expected waiting time of vehicles in the queue and in the system. We also determine the time each vehicle spends in the queue waiting for service and the mean queue length for all the channels in each section. The result shows fair traffic congestion in Calabar – highway by IBB road intersection especially in the morning and evening hours for all the locations.

Keywords: Queuing theory, Traffic congestion, traffic intensity, FIFO, System design

INTRODUCTION

Road traffic signal used as a road junction control measure to minimize congestion currently inadequate and causes traffic congestion. This is evident from the observation that although the majority of the signal lights have been revamped, they are being augmented currently by human traffic wardens (TW) to manage the situation. On many occasions have we observed activities of these wardens on traffic flow rates at signalized junctions during peak hours and their interventions amazingly make positive impact on the congestion problem. Given that roads are constantly used by vehicles at all times, human intervention for optimal road vehicular traffic control by TW cannot be avoided. The primary contribution of this paper is to formulate a model to control traffic interruption in Calabar - highway by IBB road intersection. Other possible benefit of this work is that it serves as a basis to other interesting investigations to characterize traffic congestion and the results obtained may serve as vital inputs to decisions that seek to improve traffic control and management. The objective therefore is to investigate the problem of congestion on the road segment and subsequently build upon this investigation to develop efficient tools capable of predicting and providing intelligent information on vehicular traffic flow.

Queuing theory is the mathematical study of waiting lines, or the act of joining a line (queues). In queuing theory, a model is constructed so that queue lengths and waiting times can be predicted (Sundarapandian, 2009). It is also believed that the high volume of vehicles, the inadequate infrastructure and the irrational distribution of the development are the main reasons for increasing traffic jam (Randl, 2012) Chao etal (2009) noted that traffic congestion has little or no impact on the frequency of road accidents on the motorway. If people in the developing world keep buying vehicles then simple upgrades in fuel-efficiency alone are not going to be enough to stop a steady uptick in global temperatures (Tencer, 2012). The ordered logistic models to provide insight into the human-factors aspect of the introduction of advanced technologies with respect to more sensitive segments of the driver population was developed (Yannis etal, 2010). The level of traffic congestion does not affect the severity of road crashes on the motorway. The impact of traffic flow on the severity of crashes, however, showed an interesting result (Quddu etal, 2010). An indispensable activity of our daily lives and road transport is one popular approach to mobility (Jojo, 2014). Queues are waiting lines which occur whenever units must wait for a facility because the facility may be busy and therefore it is unavailable to render service required. Whenever, the problem of congestion arises in the course of traffic management, the queuing theory and its application always comes into picture. Traffic congestion is a situation on road networks which occurs as its use increases, and is characterized by slower speeds, longer trip times and increased vehicular queuing. Congestion can also happen due to non-recurring highway incidents, such as a crash or road works, which may reduce the capacity of road below normal levels. When vehicles are fully stopped for a period of time then this phenomenon is known as traffic snarl-up. Traffic congestion can lead to drivers becoming frustrated and engaging in road rage. It occurs when a mass of traffic requires space greater than the available road capacity. There are a number of specific circumstances which aggravate congestion; most of them reduce the capacity of a road at a given point or over a certain length, or increase the number of vehicles required for a given volume of people or goods (Mala and Varma, 2016). The methods of queuing theory to solve problem of optimizing traffic light phases on signal-controlled road intersections. The flow of vehicles on multi-lane roads is described by Poisson processes (Babicheva, 2015). Road traffic management (RTM) has been very challenging and most attempts to address the problem have yielded little results.

System Design: Representation of the Calabar - Highway by IBB road intersection traffic flow

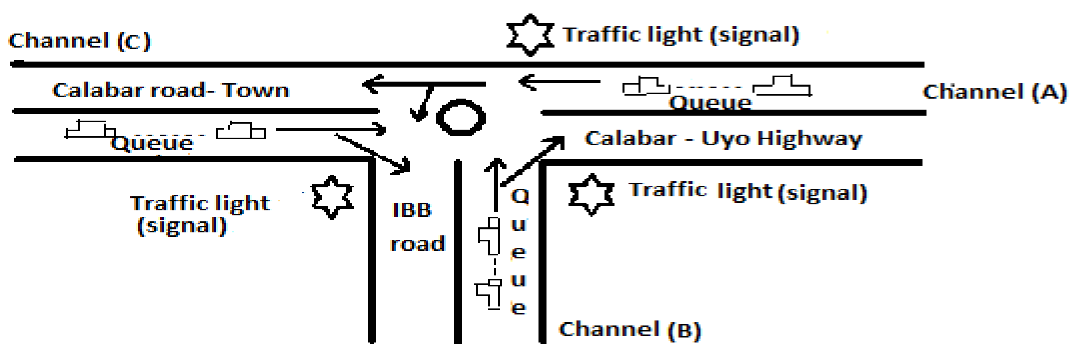


Fig 1: Typical representation of Calabar- Highway by IBB intersection

In the figure above, the round shape in the middle represents the Calabar - highway by IBB intersection and incoming arrows to the highway represents the arrivals of vehicles. Channel (A) indicates highway – Uyo road to highway - IBB intersection. Channel (B) indicates IBB road to highway – IBB intersection and channel (C) indicates Calabar road (town) to highway – IBB intersection. The three traffic signals is the system unit control by one server based on the time assigned to each channel for vehicles to pass to reduce congestion in the system.

Model Notations

λ_k = mean arrival rate (expected number of arrival per unit time)

μ_k = mean service rate (expected number of departure per unit time)

k = exact number of vehicle

π_k = probability of exactly k vehicle in the system

$N(t)$ = number of vehicle in the system at time $t \geq 0$

$E[N(t)]$ = expected number of vehicles in the system

$V[N(t)]$ = variance of vehicles in the system

$P[\pi + \tau \leq t]$ = distribution of waiting time in the system

$E[H]$ = average waiting time in the system
$V[H]$ = variance waiting time in the system
$E[N_q(t)]$ = expected number of vehicles waiting in the queue for service
$V[N_q(t)]$ = variance of vehicles waiting in the queue for service
$P[\pi \leq t]$ = distribution of waiting time in the queue
$E[W]$ = expected waiting time in the queue
$V[W]$ = variance waiting time in the queue
F_T = time spent in the queue waiting for service
L_M = mean queue length

Assumptions

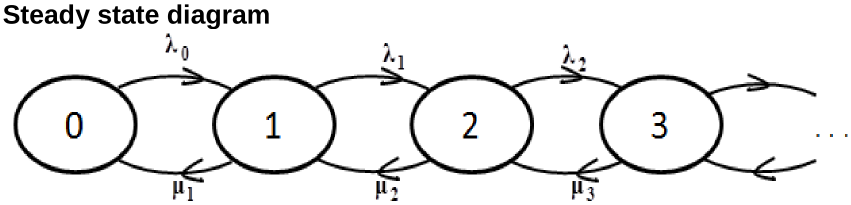
(a) $N(t) = k$ pdf of arrival time is $\ell^{\lambda k}, \ k = 0, 1, 2 \ldots$
(b) $N(t) = k$, pdf of service completion is $\ell^{\lambda k}, \ k = 1, 2 \ldots$
(c) The queuing discipline is First in First out (FIFO)
(d) The number of vehicle in the system is very large
(e) Single vehicle consume a very small percentage of the system resources
(f) Total number of car drivers or motorist on the highway is very large
(g) All random variables are assumed to be independent

METHODOLOGY

Model formation/description

This is a single server queue with Poisson arrivals and exponential service times. Vehicles are served according to first in first out (FIFO) serve discipline and there is infinite capacity waiting room. The channel considered in this research is multiple queue single servers. The model will be formulated using steady state probability process.

Steady state diagram



Where $\lambda_k = \lambda, \quad k = 0, 1, 2, ...$ and $\mu_k = \mu \ , \quad k = 1, 2, ...$
Considering the diagram above;
Flow out of 0 $\rightarrow \lambda_0 \phi_0 = \mu_1 \phi_1 \leftarrow$ flow into o
Flow out of 1 $\rightarrow \lambda_1 \phi_1 + \mu_1 \phi_1 = \lambda_0 \phi_0 + \mu_2 \phi_2 \leftarrow$ flow into 1
Flow out of 2 $\rightarrow \lambda_2 \phi_2 + \mu_2 \phi_2 = \lambda_1 \phi_1 + \mu_3 \phi_3 \leftarrow$ flow into 2
.
Flow out of k $\rightarrow \lambda_k \phi_k + \mu_k \phi_k = \lambda_{k-1} \phi_{k-1} + \mu_{k+1} \phi_{k+1} \leftarrow$ flow into k
We solve the above flows recursively

$$\phi_1 = \frac{\lambda_0}{\mu_1} \phi_0 \ , \ \phi_2 = \frac{\lambda_1}{\mu_2} \phi_2 + \frac{1}{\mu_2} (\mu_1 \phi_1 - \lambda_0 \phi_0) \ = \frac{\lambda_1}{\mu_2} \phi_1 = \frac{\lambda_1 \lambda_0}{\mu_2 \mu_1} \phi_0 \ , \ldots \ldots ,$$

$$\phi_k = \left[\frac{\lambda_k \lambda_{k-1},....,\lambda_0}{\mu_k \mu_{k-1},....,\mu_1}\right]\phi_0 \tag{1}$$

Let $T_k = \left[\frac{\lambda_k \lambda_{k-1},....,\lambda_0}{\mu_k \mu_{k-1},....,\mu_1}\right] = \left(\frac{\lambda}{\mu}\right)^k = \rho^k \quad k=1$ and let $T_0 = 1$ (2)

Where $\rho = \frac{\lambda}{\mu}$ is the traffic intensity (or utilization factor)

Then $\phi_k = T_k \phi_0$, $k = 1,2$ (3)

and $\sum \phi_k = 1$. So $(T_0 + T_1 + T_2 + T_3 .....)$

$$\phi_0 = \left[\sum_{k=0}^{\infty} T_k\right]^{-1}$$

Recall, in calculus the series as $k \to \infty$ the sum $\sum_{k=0}^{\infty} x^k = \frac{1}{1-x}$ provided $|x| < 1$. Then

$$\sum_{k=0}^{\infty} T_k = \sum_{k=0}^{\infty}\left(\frac{\lambda}{\mu}\right)^k = \sum_{k=0}^{\infty}\rho^k = \frac{1}{1-\rho}$$ Provided $\rho < 1, \lambda < \mu$

Thus $\phi_0 = \frac{1}{[1-\rho]^{-1}} = 1-\rho$ (4)

Substituting equation (2) and (4) into (3), we have

$$\phi_k = \rho^k (1-\rho), \quad k \geq k \tag{5}$$

Derivation of expected number of vehicles in the system and its variance

Let $N(t)$ be the number of vehicles in the system (those who are waiting plus any in the service) at time t . Then

$$E[N(t)] = \sum_{k=0}^{\infty} k\phi_k \tag{6}$$

From equation (5)

$$\sum_{k=0}^{\infty} k\phi_k = \sum_{k=0}^{\infty} k\rho^k (1-\rho) = 0 + \rho(1-\rho) + 2\rho^2(1-\rho) + 3\rho^3(1-\rho) + ...$$

$$= \rho - \rho^2 + 2\rho^2 - 2\rho^3 + 3\rho^3 - 3\rho^4 \ldots = \rho + \rho^2 + \rho^3 + \rho^4 + \ldots$$

The above expression is the sum of series to infinity. From the formula is given by

$$\sum_{k=0}^{\infty} x^k = \frac{a}{1-r} , |r| < 1 , \tag{7}$$

Where a the first is term of the series and r is the common ratio. Then

$$E[N(t)] = \rho[1-\rho]^{-1}$$

Variance of expected number of vehicles in the system

$$E[N(t)]^2 = \sum_{k=0}^{\infty} k^2 \phi_k = \sum_{k=0}^{\infty} k^2 \rho^k (1-\rho) \tag{8}$$

Expanding (8) and apply equation (7) gives

$$E[N(t)]^2 = \frac{\rho(1+\rho)}{(1-\rho)^2}$$

$$V[N(t)] = E[N(t)]^2 - [E[N(t)]]^2 = \frac{\rho(1+\rho)}{(1-\rho)^2} - \frac{\rho^2}{(1-\rho)^2} = \frac{\rho}{(1-\rho)^2}$$

Expected number of vehicles waiting in the queue for service

$$E[N_q(t)] = \sum_{k=0}^{\infty}(n-1)\phi_k = \sum_{k=0}^{\infty} n\phi_k - \sum_{k=0}^{\infty}\phi_k$$

$$= \frac{\rho}{(1-\rho)} - \rho = \frac{\rho}{(1-\rho)^2}$$

The stability condition $\rho < 1$ is in order, because if $\rho > 1$, then is means that vehicles are on the average arriving faster than the rate of service.

The Distribution of Waiting Time in the Queue

Consider a vehicle that arrive when they are k unites (vehicles) in the system, the time it waits on the queue for service is the time taken to serve the k vehicles before it arrival. Because of the memory – less property of the exponential (service time) distribution, the distribution of time required for k service completion is independent of the current arrival and is a convolution of k exponential random variables, which is the special form of the gamma distribution.
Let π = time taken to serve k vehicles before it and τ = individual vehicle service time.
Then the conditional distribution of π given k vehicles before it is given by;

$$P[\pi \le t \,\mathrm{I}\, k \text{ Vehicles in the system}] = \int_0^t \frac{\mu(\mu x)^{k-1}}{(k-1)!}\ell^{-\mu x}dx, \quad k \ge 1$$

Therefore the probability of a vehicle waiting in the queue, a time less than or equal to t for service is:

$$P(\pi \le t) = \sum_{k=1}^{\infty} P(\pi \le t \,\mathrm{I}\, k)\, P[N(t) = k] + P(\pi = 0)$$

Where $P(\pi = 0)$ = probability that no vehicles in the queue which is ϕ_0

$$P(\pi \le t) = \sum_{k=1}^{\infty}\int_0^t \frac{\mu(\mu x)^{k-1}}{(k-1)!}\ell^{-\mu x}\rho^k(1-\rho)dx + (1-\rho)$$

$$P(\pi \le t) = (1-\rho)\,\rho\sum_{k=1}^{\infty}\mu\ell^{-\mu x}\int_0^t \frac{(\mu x\rho)^{k-1}}{(k-1)!}dx + (1-\rho) \quad , \rho^k = \rho^k\rho^{k-1}$$

$$P(\pi \le t) = (1-\rho)\,\rho\int_0^t \mu\ell^{-\mu x(1-\rho)}dx + (1-\rho)\,, \qquad \ell^{\mu x\rho} = \sum_{k=1}^{\infty}\frac{(\mu x\rho)^{k-1}}{(k-1)!}$$

$$\Rightarrow \rho(1-\rho)\ell^{-\mu x(1-\rho)t} + 1 - \rho = (1-\rho)\ell^{-\mu(1-\rho)t} \qquad (9)$$

So the distribution of waiting time on the queue is $P(\pi \le t) = \begin{cases} 1-\rho & for\ t = 0 \\ 1-\rho\,\ell^{-\mu(1-\rho)t} & for\ t \ge 0 \end{cases}$

The probability density function of waiting time on the queue

Differentiating equation (9) w.r.t time t

$$\frac{dP(\pi \le t)}{dt} = \mu\rho(1-\rho)\ell^{-\mu(1-\rho)t}$$

The expected waiting time in the queue is;

$$E[\pi]=\int_0^\infty t dP(\pi\le t)dt \quad (10)$$

Using product rule of integration defined by: $uv-\int_0^\infty vdu$

$$E[\pi]=\int_0^\infty t dP(\pi\le t)dt=\mu\rho(1-\rho)\left[\left.\frac{t\ell^{-(1-\rho)t}}{(1-\rho)t}\right|_0^\infty+\int_0^\infty\left(\frac{\ell^{-(1-\rho)t}}{(1-\rho)t}\right)dt\right]$$

$$E[\pi]=\frac{\rho}{\mu(1-\rho)}$$

The variance of the waiting time in the queue

$$Var[\pi]=E[\pi]^2-E[[\pi]]^2 \quad (11)$$

$$E[\pi]^2=\int_0^\infty t^2 dP(\pi\le t)dt=\int_0^\infty t^2\mu\rho(1-\rho)\ell^{-\mu(1-\rho)t}dt$$

$$\Rightarrow \mu\rho(1-\rho)\int_0^\infty t^2\ell^{-\mu(1-\rho)t}dt=-\mu\rho(1-\rho)\left[\left.\frac{t^2\ell^{-\mu(1-\rho)t}}{\mu(1-\rho)}\right|_0^\infty+\frac{2}{\mu(1-\rho)}\int_0^\infty t\ell^{-\mu(1-\rho)t}dt\right]$$

$$E[\pi]^2=\frac{2\rho}{\mu^2(1-\rho)^2}$$

Substituting equation (10) and (12) into (11), we have

$$Var[\pi]=\frac{2\rho}{\mu^2(1-\rho)^2}-\frac{\rho^2}{\mu^2(1-\rho)^2}=\frac{2\rho-\rho^2}{\mu^2(1-\rho)^2}$$

The distribution of waiting time in the system

The waiting time in the system is the time taken to service k vehicles before the arrival of particular car plus its own service time. Let Y_i be the service time of the i^{th} vehicle before it arrival, then $\pi+\tau$ is total waiting time or delay of the vehicle in the system.

$\pi+\tau=Y_1+Y_2+Y_3+\ldots+Y_{k+1}$, the sum of $k+1$ independent exponential service times with parameter μ has the gamma distribution $\Gamma(k+1)$. The conditional probability of $\pi+\tau\le t$ given that there are k vehicles before i^{th} vehicle is;

$$P[\pi+\tau\le t \text{ I } k \text{ Vehicles before } i^{th} \text{ vehicle }]=\int_0^t\frac{\mu(\mu x)^{k-1}}{k!}dx \quad (12)$$

The conditional probability of $\pi+\tau\le t$ is

$$P[\pi+\tau\le t]=\sum_{k=1}^{\infty}P[\mu+\tau\le t \text{ I } k]\,P[N(t)=k]\Rightarrow\sum_{k=1}^{\infty}\int_0^t\frac{\mu(\mu x)^k\ell^{-\mu x}}{k!}\rho^k(1-\rho)dx=\int_0^t\left(\sum_k\frac{(\mu\rho x)^k}{k!}\right)(1-\rho)\ell^{-\mu x}dx$$

$$\int_0^t\mu\ell^{-\mu\rho x}(1-\rho)\ell^{-\mu x}dx=\left.\ell^{-\mu x(1-\rho)}\right|_0^t$$

Hence, $P[\pi+\tau\le t]=1-\ell^{-\mu(1-\rho)t}$ (13)

The probability density function of $\pi+\tau$

Differentiating equation (13) w.r.t time t, we have

$$\frac{d}{dt}P[\pi+\tau\le t]=\mu(1-\rho)\ell^{-\mu(1-\rho)t} \qquad (14)$$

The average waiting time in the system

Let $\pi+\tau= \ H$ be the total waiting time. Then the expected waiting time in the system is;

$$E[H] = \int_0^\infty t\mu(1-\rho)\ell^{-\mu(1-\rho)t}dx$$

$$E[H]=\int_0^\infty t\mu(1-\rho)\ell^{-\mu(1-\rho)t}dx \qquad (15)$$

$$E[H]= -\mu(1-\rho)\left[\left.\frac{t\ell^{-\mu(1-\rho)t}}{\mu(1-\rho)}\right|_0^\infty+\frac{1}{\mu(1-\rho)}\int_0^\infty \ell^{-\mu(1-\rho)t}dx\right]$$ (Using integration by part)

$$= \frac{1}{\mu(1-\rho)}$$

Variance of the waiting time in the system

$$E[H]^2 = \int_0^\infty t^2\mu(1-\rho)\ell^{-\mu(1-\rho)t}dx \qquad (16)$$

Integrating equation (16) using nitration by part, we have

$$E[H]^2 = -\mu(1-\rho)\left[\left.\frac{t^2\ell^{-\mu(1-\rho)t}}{\mu(1-\rho)}\right|_0^\infty+\frac{2}{\mu(1-\rho)}\int_0^\infty t\ell^{-\mu(1-\rho)t}dx\right]$$

$$=\mu(1-\rho)\left[\frac{2}{\mu^2(1-\rho)^2}\right]=\frac{2}{\mu(1-\rho)}$$

$$Var[H]=E[H]^2-[E[H]]^2$$

$$=\frac{2}{\mu(1-\rho)}-\frac{1}{\mu^2(1-\rho)^2} \quad = \quad \frac{2\mu(1-\rho)-1}{\mu^2(1-\rho)^2}$$

Explanation of data at different channels to highway by IBB intersection at different time

Data collected for arrival rate and service rate of vehicles at the highway – IBB intersection for 15days in the peak hours of morning (8:00 – 10:30am), afternoon (12:00 – 2:30pm) and evening (4:00 – 6:00pm) respectively is shown in table 1 below. Also the average arrival rate and service rate is calculated for 15days which is used for computation.

Table 1: Tabular representation of the traffic flow situation at the high – way by IBB intersection

Channels	Section	Arrival Time		Service Time		Arrival rate [λ]	Service rate [μ]	Traffic intensity [ρ]
		Vehicles	Minutes	Vehicles	Minutes			
A	Morning	44	1.97	47	1.63	22	29	0.7586
B	Morning	38	2.04	43	1.51	19	28	0.6786
C	Morning	42	2.29	45	1.53	18	29	0.6207
A	Afternoon	39	2.30	45	1.24	20	36	0.5556
B	Afternoon	30	2.27	49	1.19	13	41	0.3171
C	Afternoon	38	2.29	43	1.20	17	36	0.4722
A	Evening	49	2.29	53	1.44	21	36	0.5833
B	Evening	46	2.37	48	1.28	19	38	0.5000
C	Evening	48	2.15	51	1.45	22	35	0.6286

Channel A = Highway-Uyo road, Channel B = IBB road, Channel C = Highway- Town road

Table 2: Tabular representation of the traffic congestion at the Highway – IBB intersection

Channels	Section	$E[N(t)]$	$E[N_q(t)]$	$E[\pi[$	$E[H]$	$V[W]$	$V[H]$	F_T	L_M
A	Morning	3.1425	2.3839	0.1084	0.1428	0.0188	0.2653	0.9454	1.4635
B		2.1114	1.4328	0.0754	0.1111	0.0110	0.2099	0.3985	0.9248
C		1.6364	1.0157	0.0564	0.0909	0.0071	0.1736	0.2312	0.7428
A	Afternoon	1.2502	0.6946	0.0347	0.0625	0.0031	0.1211	0.1149	2.2981
B		0.3073	0.1471	0.0113	0.0357	0.0007	0.0701	0.0365	0.1577
C		0.8947	0.4225	0.0249	0.0526	0.0020	0.1025	0.0724	1.4671
A	Evening	1.3998	0.8165	0.0389	0.0667	0.0037	0.0490	0.1405	2.9494
B		1.0000	0.5000	0.0263	0.0526	0.0021	0.1025	0.0794	1.5081
C		1.6925	1.0639	0.0286	0.0769	0.0051	0.1479	0.2038	4.4837

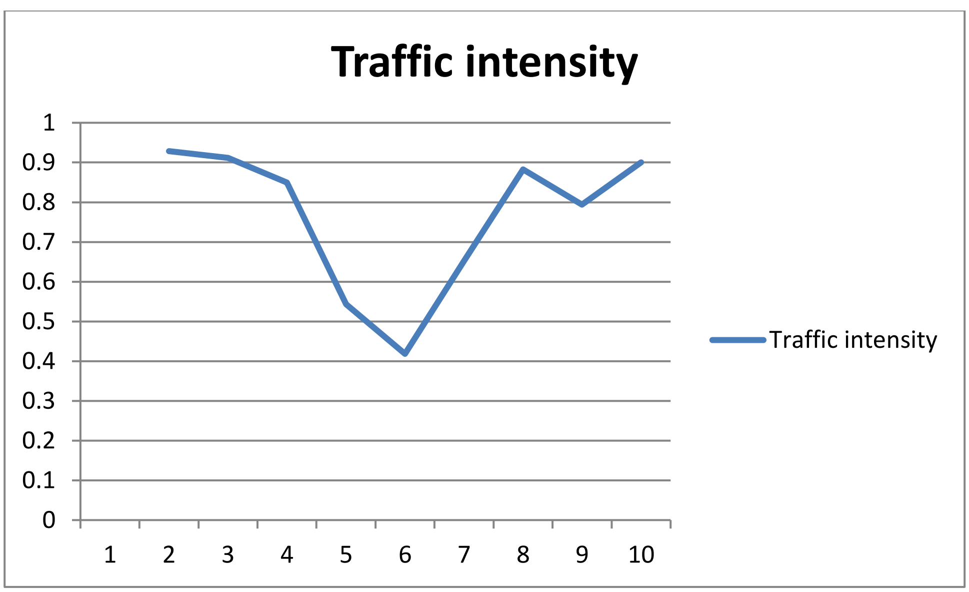


Fig 2: Graphical Representation of Traffic Intensity at Different Channels Leading to Highway by IBB Intersection

RESULTS/DISCUSSION

From table 1 above, the results for the three channels in the morning section shows fairly stable traffic congestion but not a smooth and good traffic flow. Also, the afternoon section the traffic intensity results for the three channels shows good stable and smooth traffic flow. During evening hours, the traffic intensity recorded smooth and stable traffic flow situation for the three channels intersection. The result from table 2 shows that the expected waiting time of vehicles in the system is more than the expected waiting time of vehicles in the queue for all the channels in each section per day. We also observed from the results that the expected numbers of vehicles in the system is more than the expected numbers of vehicles in the queue waiting to be served in each channel. The time each vehicle spent in the queue waiting for service in all the channels per session is less than the mean queue length. From fig 2; the graph explained the traffic flow intensity and its shows unstable traffic congestion in the morning and evening than the afternoon section.

CONCLUSION

Based on the results obtained, it shows that the traffic situation in channel C (Highway – town) with traffic intensity of 0.6207 indicate a stable flow of vehicles than channel A (Highway – Uyo) and B (IBB - road) in the morning section. During the afternoon section, the traffic situation is fantastic, meaning that there is no traffic congestion problem in the entire channels especially channel B (IBB - road) and channel C (Highway – town) respectively. Also in the evening hours, channel B (IBB – road) shows smooth traffic flow with the traffic intensity of 0.5000, seconded by channel A (Highway – Uyo) with traffic intensity of 0.5833 and channel C (Highway – town) with traffic intensity of 0.6286 intersection. We also observed that the time each vehicle spent in the queue is less than the length of vehicles in the system in the entire channels per section. Generally, the traffic congestion at Highway – Uyo, IBB – road and highway – town intersection in Calabar is controllable which does not have serious effect on the flow of vehicles.

REFERENCE

Babicheva, T.S. (2015). The Use of Queuing Theory at Research and Optimization of Traffic onthe Signal-controlled RoadIntersections. Procedia Computer Science, 55, 469-478.

Chao, W., Mohammed, A. Q and Stephen, G. I. (2009). Impact of Traffic Congestion on Road Safety: A Spatial Analysis of the M25 Motorway in England. Accident analysis and prevention. Publisher: Elsevier Ltd, 41(4); 798-808.

Jojo, D. L. (2014). Predicting Traffic Congestion: A Queuing Perspective. Open Journal of Modelling and Simulation, 2, 57-66.

Mala, S and Varma, P. (2016). Minimization of Traffic Congestion by Using Queuing Theory. IOSR Journal of Mathematics (IOSR-JM), 6(4); 436 – 448.

Sundarapandian, V. (2009). Queuing Theory: Probability, Statistics and Queuing Theory. PHI Learning. ISBN 8120338448.

Quddus, M. A, Chao, W and Stephen, G. I. (2010). Road Traffic Congestion and Crash Severity: Econometric Analysis Using Ordered Response Models. Journal of Transportation Engineering.136 (5); 424-435.

Randal, O. (2012). Solving the problem of traffic congestion. Retrieved January 16, 2013, from http://www.ncpa.org/pub/ba773.

Tencer, D. (2011). Number of Cars Worldwide Surpasses 1 Billion; Retrieved February 07, 2013 from http://www.huffingtonpost.ca/2011/08/23/car-population_n_934291.html.

Yannis, G., Antoniou, C., Vardaki, S and Kanellaidis, G. (2010). Older Drivers' Perception and Acceptance of In-Vehicle Devices for Traffic Safety and Traffic Efficiency. Journal of Transportation Engineering, 136(5); 472-479.

Accepted 27 July, 2017

Citation: Akra, UP, Ntekim OE (2017). Exploring Queuing Theory to Minimize Traffic Congestion Problem in Calabar-Highway by IBB Road, Cross River State. International Journal of Statistics and Mathematics, 4(1): 099-107.