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Research Article

A Generalization of Minimax Distribution

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In this paper, we have introduced the new family of distribution called Exponentiated Minimax distribution (EMD). We have studied some statistical properties of this distribution that including the moments, moment generating function, characteristic function, median, mode, harmonic mean, entropy, reliability function, hazard function and the reverse hazard function. Also the unknown parameter λ of this distribution is estimated by the method of maximum likelihood.

Keywords: Minimax distribution, Exponentiated Minimax distribution, Structural properties and Maximum Likelihood Estimation.

INTRODUCTION

The Minimax probability distribution was originally proposed by McDonald (1984). Jones (2007) explored the genesis of the Minimax distribution and made some similarities between Beta and Minimax distributions. The Minimax distribution denoted by $\text{Minimax}(\alpha,\theta)$ is a family of continuous probability distribution defined on the interval $[0,1]$ with probability density function (pdf) given by;

$$g(x;\theta,\alpha)=\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1} \quad ; 0<x<1\ , \quad \theta\alpha>0 \qquad (1.1)$$

The corresponding cumulative density function (cdf) is given by;

$$G(x;\theta,\alpha)=1-\ (1-x^{\alpha})^{\theta} \quad ; 0<x<1\ , \quad \theta\alpha>0 \qquad (1.2)$$

where α and θ are two shape parameters. The minimax density is unimodal, uniantimodal, increasing, decreasing or constant depending on the values of its parameters. Minimax distribution is a special case of generalized beta distribution.

The Exponentiated distributions have been studied widely in statistics since 1995 and a number of authors have developed various classes of these distributions; Mudholkar et al (1995) proposed the exponentiated Weibull distribution. Gupta et al. (1998) first proposed a generalization of the standard exponential distribution, called the exponentiated exponential (*EE*) distribution. Nadarajah and Kotz (2003) defined and studied the exponentiated Fr´echet distribution, and Nadarajah (2005) defined and studied the exponentiated Gumbel distribution. Barreto-Souza and Cribari-Neto (2009) developed the exponentiated exponential-Poisson distribution, whereas Silva et al. (2010) proposed the exponentiated exponential-geometric distribution. Lemonte and Cordeiro (2011) introduced the exponentiated generalized inverse Gaussian distribution. Cordeiro et al. (2013) proposed the beta exponentiated Weibull distribution, whereas Adepoju, K.A Chukwu A.U and Shittu, O.I (2014) defined and studied the exponentiated nakagami distribution. Oguntunde P. E (2015) introduced the exponentiated weighted exponential distribution. Recently, Fatima and Ahmad (2016) considered the characterization and bayesian estimation of Minimax distribution. Here, in the same way, we have generalized the Minimax distribution.

EXPONENTIATED MINIMAX DISTRIBUTION

The Exponentiated family of distribution is derived by powering a positive real number to the cumulative distribution function (CDF) of an arbitrary parent distribution by a shape parameter say; $\lambda > 0$. Its pdf is given by;

$$f(x)=\lambda g(x)[G(x)]^{\lambda-1} \qquad (2.1)$$

The corresponding cumulative distribution function (cdf) is given by

$$F(x)=[G(x)]^{\lambda},\ \lambda>0 \qquad (2.2)$$

With this understanding, we insert Equations (1.1) and (1.2) into Equation (2.1) to give the pdf of the Exponentiated Minimax distribution as;

$$f(x)=\alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\left[1-(1-x^{\alpha})^{\theta}\right]^{\lambda-1};\ 0<x<1,\ \theta,\alpha,\lambda>0 \qquad (2.3)$$

The graphs of density function are plotted for various values of α, θ *and* λ are given in figure (1.1)

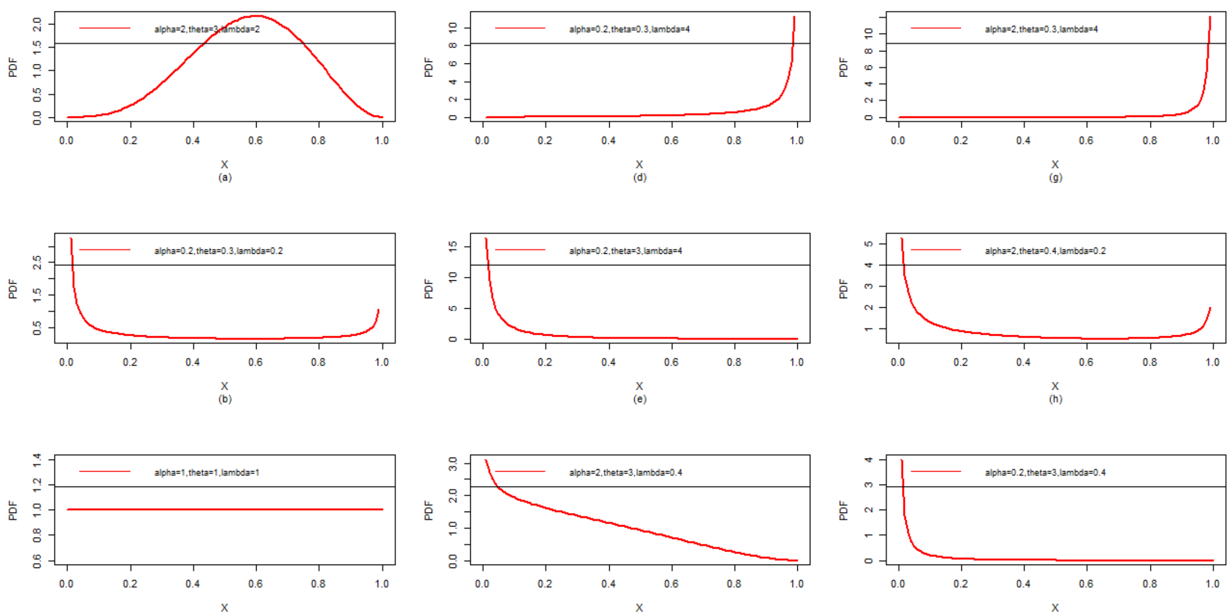

Figure1.1 Exponentiated Minimax densities for selected values of (α, θ and λ) ;(a) $\alpha=2, \theta=3$ and $\lambda=2$; (b) $\alpha=0.2, \theta=0.3$ and $\lambda=0.2$; (c) $\alpha=1, \theta=1$ and $\lambda=1$; (d) $\alpha=0.2, \theta=0.3$ and $\lambda=4$; (e) $\alpha=0.2, \theta=3$ and $\lambda=4$;(f) $\alpha=2, \theta=3$ and $\lambda=0.4$; (g) $\alpha=2, \theta=0.3$ and $\lambda=0.4$;(h) $\alpha=2, \theta=0.4$ and $\lambda=0.2$; (i) $\alpha=0.2, \theta=3$ and $\lambda=0.4$.

Figure 1.1 illustrates some of the possible shapes of Exponentiated Minimax distribution for different values of the parameters α , θ and λ. Figure 1.1 shows that the density function of Exponentiated Minimax are unimodal, uniantimodal, increasing, decreasing or constant depending on the values of parameters.
The corresponding (cdf) of the Exponentiated Minimax distribution is given by

$$F(x)=\left[1-(1-x^{\alpha})^{\theta}\right]^{\lambda} \qquad (2.4)$$

Where α , θ and λ are the shape parameters

The plots for the cdf of the proposed model at various values of parameters $\alpha, \theta \, and \, \lambda$ is shown in Figure 1.2

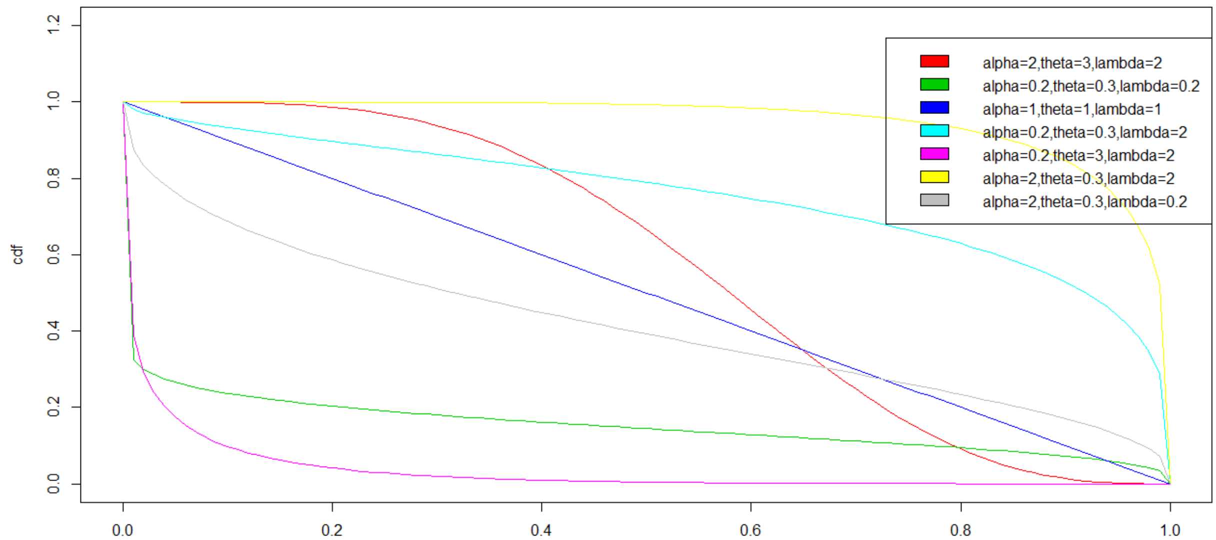

Fig.1.2: distribution function of(EMD)under various values of its Parameters

Special Cases

Case 1: When $\lambda = 1$, then Exponentiated Minimax distribution (2.3) reduces to Minimax distribution (MD) with probability density function as:

$$f(x) = \alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1} \quad ; 0 < x < 1 \, , \quad \theta\alpha > 0 \qquad (3.1)$$

Case 2: When $\lambda = \alpha = \theta = 1$, then Exponentiated Minimax distribution (2.3) reduces to Uniform distribution (UD) with probability density function as:

$$f(x) = 1 \, ; 0 < x < 1 \qquad (3.2)$$

Case 3: When $\theta = \lambda = 1$, then Exponentiated Minimax distribution (2.3) reduces to Power distribution (PD) with probability density function as:

$$f(x;\alpha) = \alpha \; x^{\alpha-1}; \quad 0 < x < 1 \;\; , \alpha > 0 \qquad (3.3)$$

Case 4: When $\alpha = \lambda = 1$, then Exponentiated Minimax distribution (2.3) reduces to one parameter Minimax distribution with probability density function as:

$$f(x;\theta) = \theta \; (1-x)^{\theta-1}; \quad 0 < x < 1 \quad , \theta > 0 \qquad (3.4)$$

Reliability Analysis

(i) Reliability function R(x)

The reliability function or survival function $R(x)$ .This function can be derived using the cumulative distribution function and is given by

$$R(x) = (1 - F(x))$$

$$R(x) = 1 - \left\{1 - (1-x^{\alpha})^{\theta}\right\}^{\lambda} ; 0 < x < 1 \qquad (4.1)$$

The corresponding plot for the reliability function at various values of $\alpha, \theta \, and \, \lambda$ is shown in Figure 2.1.

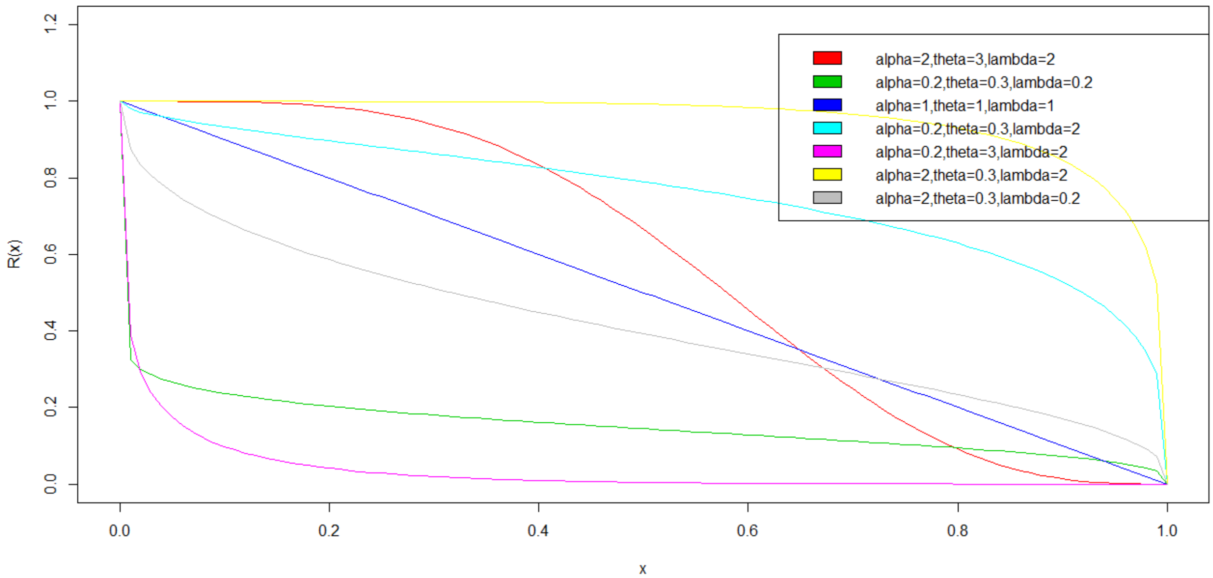

Fig.2.1: Reliability function of(EMD)under various values of its Parameters

(ii) Hazard Function H(x)

The hazard or instantaneous rate function is denoted by $H(x)$. The hazard function of x can be interpreted as instantaneous rate or the conditional probability density of failure at time x, given that the unit has survived until x. The hazard function is defined to be

$$H(x)=\frac{f(x)}{R(x)}$$

[illegible] (4.2)

(iii) Reverse Hazard function $\phi(x)$

The reverse hazard function can be interpreted as an approximate probability of failure in $[x, x+d]$, given that the failure had occurred in $[0, x]$. The reverse hazard function $\phi(x)$ is defined to be

[illegible]

[illegible] (4.3)

Structural Properties of Exponentiated Minimax Distribution:

Theorem 5.1 Let $X=(X_1, X_2, \ldots, X_n)$ be a random sample of size n from the Exponentiated Minimax distribution with probability density function

$$f(x)=\lambda\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\left[1-(1-x^{\alpha})^{\theta}\right]^{\lambda-1};\quad 0<x<1$$

Then

$$E(X^{r})=\lambda\,\Gamma\left(\frac{r}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^{j}}{\Gamma\left(\frac{r}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\qquad r=1,2,\ldots$$

Proof: Since we know that the r[th] moment of a random variable x is given by

$$E(X^{r})=\int_{0}^{1}x^{r}f(x)\,dx \tag{5.1}$$

Now using eq. (2.3) in eq. (5.1), we have

$$E(X^{r})=\int_{0}^{1}x^{r}\lambda\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\left[1-(1-x^{\alpha})^{\theta}\right]^{\lambda-1}dx$$

Put $1-(1-x^{\alpha})^{\theta}=t$; as $x\to 0,\ t\to 0$; as $x\to 1,\ t\to 1$,

$\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1}dx=dt$ and $x=\left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{\frac{1}{\alpha}}$

$$E(X^{r})=\lambda\int_{0}^{1}\left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{\frac{r}{\alpha}+1-1}t^{\lambda-1}dt \tag{5.2}$$

We apply the series expansion

$$(1-z)^{b-1}=\sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma b}{\Gamma(b-j)\,j!}\,z^{j}$$

Similarly, $$\left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{\frac{r}{\alpha}+1-1}=\sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right)j!}\ (1-t)^{\frac{j}{\theta}} \tag{5.3}$$

By using (5.3) in (5.2), we get

$$E(X^{r})=\lambda\sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right)j!}\int_{0}^{1}t^{\lambda-1}(1-t)^{\frac{j}{\theta}+1-1}dt$$

On solving the above equation, we get

$$E(X^{r})=\lambda\,\Gamma\left(\frac{r}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^{j}}{\Gamma\left(\frac{r}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \tag{5.4}$$

Moments Of Exponentiated Minimax Distribution:

If we put r=1 in eq. (5.4), we get the mean of Exponentiated Minimax distribution which is given by

$$\mu_1' = \lambda\,\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) \qquad (5.5)$$

If we put r=2 in eq. (5.4), we have

$$\mu_2' = \lambda\,\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) \qquad (5.6)$$

Thus the variance of Exponentiated Minimax distribution is given by

$$\mu_2 = \lambda\,\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) - \left\{\lambda\,\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2 \qquad (5.7)$$

If we put r=3 in eq. (5.4), we have

$$\mu_3' = \lambda\,\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) \qquad (5.8)$$

Thus $$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 \qquad (5.9)$$

After substituting the values of eq. (5.5), eq. (5.6) and eq. (5.8) in eq. (5.9), we have

$$\mu_3 = \lambda\,\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) - 3\lambda\,\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right)$$

$$\times\,\lambda\,\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) + 2\left\{\lambda\,\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^3 \qquad (5.10)$$

If we put r=4 in eq. (5.4), we have

$$\mu_4' = \lambda\,\Gamma\left(\frac{4}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{4}{\alpha}+1-j\right)j!}\,B\left(\lambda,\frac{j}{\theta}+1\right) \qquad (5.11)$$

Thus, $$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 \qquad (5.12)$$

After substituting the values of eq. (5.5), eq. (5.6), eq. (5.8) and eq. (5.11) in eq. (5.12), we have

$$\mu_4 = \lambda\Gamma\left(\frac{4}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{4}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) - \lambda 4\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)$$

$$\times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\} + 6\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)$$

$$\times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2 - 3\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^4 \quad (5.13)$$

Coefficient Of Variation

It is the ratio of standard deviation and mean. Usually it is denoted by C.V. and is given by

$$C.V = \frac{\sigma}{\mu} \quad (5.14)$$

By using the value of eq. (5.5) and eq. (5.7) in eq. (5.14), we get

$$C.V = \frac{\sqrt{\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) - \left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2}}{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)} \quad (5.15)$$

Skewness And Kurtosis Of Minimax Distribution:

(i) Skewness: The most popular way to measure the Skewness and kurtosis of a distribution function rests upon ratios of moments. Lack of symmetry of tails (about mean) of frequency distribution curve is known as Skewness. The formula for measure of Skewness given by Karl Pearson in terms of moments of frequency distribution is given by

$$\beta_1 = \frac{\mu_3^2}{\mu_2^3} \quad (5.16)$$

After using eq. (5.7) and eq. (5.10) in eq. (5.16), we have

$$\beta_1 = \frac{\left[\begin{array}{l} \lambda\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-3\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)+2\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^3 \end{array}\right]^2}{\left[\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2\right]^3}$$

$$\gamma_1 = \sqrt{\beta_1} = \frac{\left\{\begin{array}{l} \lambda\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-3\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)+2\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^3 \end{array}\right\}}{\left[\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2\right]^{3/2}} \qquad (5.17)$$

(ii) Kurtosis: Kurtosis is the degree of peakedness of a distribution, defined as normalized form of the fourth central moment μ_4 of a distribution. There are several flavors of kurtosis commonly encountered, including the kurtosis proper, denoted β^2 and defined by

$$\beta_2 = \frac{\mu_4}{\mu_2^2} \qquad (5.18)$$

After using eq. (5.7) and eq. (5.13) in eq. (5.18), we have

$$\beta_2 = \frac{\left\{\begin{array}{l} \lambda\Gamma\left(\frac{4}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{4}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\lambda 4\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}+6\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2-3\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^4 \end{array}\right\}}{\left[\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2\right]^2}$$

(5.19)

$$\gamma_2 = \beta_2 - 3$$

Using equation (5.19)

$$\gamma_2 = \frac{\left\{\begin{array}{l} \lambda\Gamma\left(\frac{4}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{4}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\lambda 4\Gamma\left(\frac{3}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{3}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}+6\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right) \\ \times\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2-3\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^4 \end{array}\right\}}{\left[\lambda\Gamma\left(\frac{2}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{2}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)-\left\{\lambda\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}^2\right]^2}-3$$

(5.20)

Harmonic mean of Exponentiated Minimax distribution

The harmonic mean (H) is given as:

$$\frac{1}{H} = E\left(\frac{1}{X}\right) = \int_0^1 \frac{1}{x}\ f(x)dx$$

$$\frac{1}{H} = \int_0^1 \frac{1}{x}\ \alpha\theta\lambda\, x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\left\{1-(1-x^{\alpha})^{\theta}\right\}^{\lambda-1} dx$$

Put $1-(1-x^{\alpha})^{\theta} = t$; as $x \to 0, t \to 0$; as $x \to 1, t \to 1$

$$\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1}dx = dt \quad \text{and} \quad x = \left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{\frac{1}{\alpha}}$$

$$\frac{1}{H} = \lambda\int_0^1 \left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{1-\frac{1}{\alpha}-1} t^{\ \lambda-1}dt$$

Using the series expansion

$$\left(1-\ (1-t)^{\frac{1}{\theta}}\right)^{1-\frac{1}{\alpha}-1} = \sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(1-\frac{1}{\alpha}-j\right)j!}\ (1-t)^{\frac{j}{\theta}}$$

$$\Rightarrow \frac{1}{H} = \lambda\sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(1-\frac{1}{\alpha}-j\right)j!}\int_0^1 t^{\ \lambda-1}(1-t)^{\frac{j}{\theta}+1-1}dt$$

$$\Rightarrow \frac{1}{H} = \lambda\sum_{j=0}^{\infty}\frac{(-1)^{j}\,\Gamma\left(1-\frac{1}{\alpha}\right)}{\Gamma\left(1-\frac{1}{\alpha}-j\right)j!}B\left(\lambda, \frac{j}{\theta}+1\right) \qquad (6.1)$$

Mode and Median of Exponentiated Minimax distribution

Median of Exponentiated Minimax distribution

Theorem7.1 The approximate median m of the Exponentiated minimax distribution is given by

$$m = \left\{1-\left(1-2^{-1/\lambda}\right)^{1/\theta}\right\}^{1/\alpha} \qquad (7.1)$$

Proof. The median *m* can be usually found by using the following:

$$F(m)=P(X\le m)=\int_o^m f(x)dx=\frac{1}{2}$$

$$\Rightarrow F(m)=\int_0^m \alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\left\{1-(1-x^{\alpha})^{\theta}\right\}^{\lambda-1}dx=\frac{1}{2}$$

$$put\ \ 1-(1-x^{\alpha})^{\theta}=y \qquad ;as\ \ x\to 0, y\to 0$$
$$\alpha\theta x^{\alpha-1}(1-x^{\alpha})^{\theta-1}dx=dy \qquad ;as\ \ x\to m, y\to 1-(1-m^{\alpha})^{\theta} \qquad \Rightarrow \qquad \lambda\int_0^{1-(1-m^{\alpha})^{\theta}} y^{\lambda-1}dy=\frac{1}{2}$$

$$\Rightarrow \qquad \left[y^{\lambda}\right]_0^{1-(1-m^{\alpha})^{\theta}}=\frac{1}{2}$$

$$\Rightarrow \qquad \left\{1-(1-m^{\alpha})^{\theta}\right\}^{\lambda}=\frac{1}{2}$$

Applying log on both sides, we have

$$\lambda\log\left\{1-(1-m^{\alpha})^{\theta}\right\}=\log\left(\frac{1}{2}\right)$$

$$\Rightarrow \qquad \log\left\{1-(1-m^{\alpha})^{\theta}\right\}=-\frac{1}{\lambda}\log 2$$

$$\Rightarrow \qquad \left\{1-(1-m^{\alpha})^{\theta}\right\}=e^{\log 2^{-1/\lambda}}$$

$$\Rightarrow \qquad 1-(1-m^{\alpha})^{\theta}=2^{-1/\lambda}$$

$$\Rightarrow \qquad (1-m^{\alpha})^{\theta}=1-2^{-1/\lambda}$$

$$\Rightarrow \qquad (1-m^{\alpha})=\left(1-2^{-1/\lambda}\right)^{1/\theta}$$

$$\Rightarrow \qquad m=\left\{1-\left(1-2^{-1/\lambda}\right)^{1/\theta}\right\}^{1/\alpha} \tag{7.2}$$

Mode of Exponentiated Minimax distribution

Mode=3Median -2Mean (7.3)

Substituting the values of eq. (7.2) and eq. (5.5) in eq. (7.3), we have

$$\text{Mode}=3\left\{1-\left(1-2^{-1/\lambda}\right)^{1/\theta}\right\}^{1/\alpha}-2\left\{\lambda\,\Gamma\left(\frac{1}{\alpha}+1\right)\sum_{j=0}^{\infty}\frac{(-1)^j}{\Gamma\left(\frac{1}{\alpha}+1-j\right)j!}B\left(\lambda,\frac{j}{\theta}+1\right)\right\}$$

Moment generating function and characteristic function

Theorem8.1 Let X have a Exponentiated Minimax distribution. Then moment generating function of X denoted by $M_X(t)$ is given by:

$$M_X(t) = E(e^{tx}) = \lambda \sum_{r=0}^{\infty}\sum_{j=0}^{\infty} \frac{t^r}{r!} \frac{(-1)^j \Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right) j!} B\left(\lambda, \frac{j}{\theta}+1\right) \qquad (8.1)$$

Proof: By definition

$$M_X(t) = E(e^{tx}) = \int_0^1 e^{tx} f(x)dx$$

Using Taylor series

$$M_X(t) = \int_0^1 \left(1 + tx + \frac{(tx)^2}{2!} + \cdots\right) f(x)dx$$

$$\Rightarrow \qquad M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} \int_0^1 x^r f(x)dx$$

$$\Rightarrow \qquad M_X(t) = \sum_{r=0}^{\infty} \frac{t^r}{r!} E(X^r)$$

$$\Rightarrow \qquad M_X(t) = \lambda \sum_{r=0}^{\infty}\sum_{j=0}^{\infty} \frac{t^r}{r!} \frac{(-1)^j \Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right) j!} B\left(\lambda, \frac{j}{\theta}+1\right)$$

This completes the proof.

Theorem 8.2 Let X have a Exponentiated Minimax distribution. Then characteristic function of X denoted by $\phi_X(t)$ is given by:

$$\phi_X(t) = E(e^{itx}) = \lambda \sum_{r=0}^{\infty}\sum_{j=0}^{\infty} \frac{(it)^r}{r!} \frac{(-1)^j \Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right) j!} B\left(\lambda, \frac{j}{\theta}+1\right) \qquad (8.2)$$

Proof: By definition

$$\phi_X(t) = E(e^{itx}) = \int_0^1 e^{itx} f(x)dx$$

Using Taylor series

$$\phi_X(t) = \int_0^1 \left(1 + itx + \frac{(itx)^2}{2!} + \cdots\right) f(x)dx$$

$$\Rightarrow \qquad \phi_X(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} \int_0^1 x^r f(x)dx$$

$$\Rightarrow \qquad \phi_X(t) = \sum_{r=0}^{\infty} \frac{(it)^r}{r!} E(X^r)$$

$$\Rightarrow \qquad \phi_X(t) = \lambda \sum_{r=0}^{\infty} \sum_{j=0}^{\infty} \frac{(it)^r}{r!} \frac{(-1)^j \Gamma\left(\frac{r}{\alpha}+1\right)}{\Gamma\left(\frac{r}{\alpha}+1-j\right) j!} B\left(\lambda, \frac{j}{\theta}+1\right)$$

This completes the proof.

Shannon's entropy of Exponentiated Minimax distribution

The Shannon entropy of a random variable X is a measure of the uncertainty and is given by $E\left[-\log\left(f(x)\right)\right]$, where $f(x)$ is the probability function of the random variable X. Shannon entropy of Exponentiated Minimax distribution are obtained as:

$$H(x) = -E[\log f(x)] = -E[\log\{\alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1}\}]$$
$$= -\log[\alpha\theta\lambda] - (\alpha-1)E\left(\log x\right) - (\theta-1)E\left(\log(1-x^{\alpha})\right) - (\lambda-1)E[\log 1-(1-x^{\alpha})^{\theta}]$$

$$H(x) = -\log[\alpha\theta\lambda] - (\alpha-1)I_1 - (\theta-1)I_2 - (\lambda-1)I_3 \qquad (9.1)$$

Now, $I_1 = E\left(\log x\right) = \int_0^1 \log x\{\alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1} dx$

$$= \theta\lambda \int_0^1 \log x^{\alpha}\{x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1} dx \qquad (9.2)$$

put $x^{\alpha} = y$; as $x \to 0, y \to 0$

$x^{\alpha-1} dx = \frac{dy}{\alpha}$; as $x \to 1, y \to 1$

$$= \frac{\theta\lambda}{\alpha} \int_0^1 \log y (1-y)^{\theta-1}\{1-(1-y)^{\theta}\}^{\lambda-1} dy \qquad (9.3)$$

We apply the series expansion

$$\{1-(1-y)^{\theta}\}^{\lambda-1} = \sum_{j=0}^{\infty} \frac{(-1)^j \Gamma\lambda}{\Gamma(\lambda-j)\, j!} (1-y)^{j\theta} \qquad (9.4)$$

Substitute the value of (9.4) in (9.3) we get,

$$= \frac{\theta\lambda}{\alpha} \sum_{j=0}^{\infty} \frac{(-1)^j \; \Gamma\lambda}{\Gamma(\lambda-j)\, j!} \int_0^1 y^{1-1} (1-y)^{j\theta+\theta-1} \log y dy \qquad (9.5)$$

We know that

$$\int_0^1 y^{a-1}(1-y)^{b-1} \log y dy = B(a,b)\left(\psi(a) - \psi(a+b)\right)$$

On solving the equation (9.5), we get

$$\Rightarrow I_1 = \frac{\theta\lambda}{\alpha}\sum_{j=0}^{\infty}\frac{(-1)^j)\Gamma\lambda}{\Gamma(\lambda-j)\,j!}B(1, j\theta+\theta)\big(\psi(1)-\psi(j\theta+\theta+1)\big) \qquad (9.6)$$

$$I_2 = E\big(\log(1-x^{\alpha})\big) = \int_0^1 \log(1-x^{\alpha})[\alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1}]dx \quad (9.7)$$

$$put\ \ x^{\alpha} = y \qquad ;as\ \ x\to 0, y\to 0$$
$$\alpha\, x^{\alpha-1}\,dx = dy \qquad ;as\ \ x\to 1, y\to 1$$

$$= \theta\lambda\int_0^1 \log(1-y)(1-y)^{\theta-1}\{1-(1-y)^{\theta}\}^{\lambda-1}dy \qquad (9.8)$$

We apply the series expansion

$$\{1-(1-y)^{\theta}\}^{\lambda-1} = \sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}(1-y)^{j\theta} \qquad (9.9)$$

Substitute the value of (9.9) in (9.8) we get,

$$= \theta\lambda\sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}\int_0^1 y^{1-1}(1-y)^{j\theta+\theta-1}\log(1-y)dy \qquad (9.10)$$

We know that

$$\int_0^1 y^{a-1}(1-y)^{b-1}\log(1-y)dy = B(a,b)\big(\psi(b)-\psi(a+b)\big)$$

On solving the equation (9.10), we get

$$\Rightarrow I_2 = \theta\lambda\sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}B(1, j\theta+\theta)\big(\psi(j\theta+\theta)-\psi(j\theta+\theta+1)\big) \qquad (9.11)$$

$$\Rightarrow I_3 = E\Big(\log\big(1-(1-x^{\alpha})^{\theta}\big)\Big) = \int_0^1 \log\big(1-(1-x^{\alpha})^{\theta}\big)[\alpha\theta\lambda x^{\alpha-1}(1-x^{\alpha})^{\theta-1}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1}]dx$$
(9.12)

$$put\ \ 1-(1-x^{\alpha})^{\theta} = y \qquad ;as\ \ x\to 0, y\to 0$$
$$\alpha\theta(1-x^{\alpha})^{\theta-1}x^{\alpha-1}\,dx = dy \qquad ;as\ \ x\to 1, y\to 1$$

$$= \lambda\int_0^1 \log y\; y^{\lambda-1}(1-y)^{1-1}dy \qquad (9.13)$$

On solving the above equation (9.13), we get

$$\Rightarrow I_3 = \lambda B(\lambda,1)\big(\psi(\lambda)-\psi(\lambda+1)\big) = \frac{\lambda\Gamma\lambda\Gamma 1}{\Gamma(\lambda+1)}\big(\psi(\lambda)-\psi(\lambda+1)\big)$$

$$\Rightarrow I_3 = \big(\psi(\lambda)-\psi(\lambda+1)\big) \qquad (9.14)$$

Substitute the values of (9.6), (9.11) and (9.14) in equation (9.1)

$$H(x) = -\log[\alpha\theta\lambda] - \frac{(\alpha-1)\theta\lambda}{\alpha}\sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}B(1, j\theta+\theta)\big(\psi(1)-\psi(j\theta+\theta)\big) - (\theta-1)$$

$$\times\,\theta\lambda\sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}B(1, j\theta+\theta)\big(\psi(j\theta+\theta)-\psi(j\theta+\theta+1)\big) - (\lambda-1)\big(\psi(\lambda)-\psi(\lambda+1)\big)$$

$$\Rightarrow H(x) = \theta\lambda\sum_{j=0}^{\infty}\frac{(-1)^j\,\Gamma\lambda}{\Gamma(\lambda-j)\,j!}B(1, j\theta+\theta)\left\{\left(\frac{1}{\alpha}-1\right)\big(\psi(1)-\psi(j\theta+\theta+1)\big) + (1-\theta)\big(\psi(j\theta+\theta)-\psi(j\theta+\theta+1)\big)\right\}$$

$$-\left\{(\lambda-1)\big(\psi(\lambda)-\psi(\lambda+1)\big)+\log[\alpha\theta\lambda]\right\} \qquad (9.15)$$

Order Statistics

Let $X_{(1)}$ denote the smallest of $\{X_1, X_2, \ldots, X_n\}$, $X_{(2)}$ denote the second smallest of $\{X_1, X_2, \ldots, X_n\}$, and similarly $X_{(k)}$ denote the k^{th} smallest of $\{X_1, X_2, \ldots, X_n\}$. Then the random variables $X_{(1)}, X_{(2)}, \ldots, X_{(n)}$, called the order statistics of the sample $\{X_1, X_2, \ldots, X_n\}$, has probability density function of the k^{th} order statistic, $X_{(k)}$, as:

$$f(x) = \frac{n!}{(k-1)!(n-k)!}f(x)F(x)^{k-1}[1-F(x)]^{n-k} \qquad (10.1)$$

For $k = 1, 2, \ldots, n$.

The pdf of the k^{th} order statistic is defined as:

$$f_X(k) = \frac{n!}{(k-1)!(n-k)!}\alpha\lambda\theta\, x^{\alpha-1}(1-x^\alpha)^{\theta-1}\left\{1-(1-x^\alpha)^\theta\right\}^{\lambda-1}[1-(1-x^\alpha)^\theta]^{\lambda(k-1)}$$

$$[1-\{(1-(1-x^\alpha)^\theta\}^\lambda]^{n-k}$$

$$f_X(k) = \frac{n!}{(k-1)!(n-k)!}\alpha\lambda\theta\, x^{\alpha-1}(1-x^\alpha)^{\theta-1}[1-(1-x^\alpha)^\theta]^{\lambda k-1}[1-\{(1-(1-x^\alpha)^\theta\}^\lambda]^{(n-k)}$$

$$(10.2)$$

For $k = n$ in (10.2)

The pdf of the largest statistics $X_{(n)}$ is therefore:

$$f_X(n) = n\alpha\theta\lambda\, x^{\alpha-1}(1-x^\alpha)^{\theta-1}[1-(1-x^\alpha)^\theta]^{\lambda n-1} \qquad (10.3)$$

For $k = 1$ in (10.2)

and the pdf of the smallest order statistic $X_{(1)}$ is given by:

$$f_X(1) = n\alpha\theta\lambda x^{\alpha-1}(1-x^\alpha)^{\theta-1}[1-(1-x^\alpha)^\theta]^{\lambda-1}[1-\{(1-(1-x^\alpha)^\theta\}^\lambda]^{(n-1)} \qquad (10.4)$$

Maximum likelihood Estimation for the shape Parameter λ of Exponentiated Minimax distribution assuming shape parameters α and θ are to be known:

Let us consider a random sample $\underline{x} = (x_1, x_2, \ldots, x_n)$ of size n from the Exponentiated Minimax Distribution. Then the likelihood function for the given sample observation is

$$L(\underline{x}/\lambda)=\alpha^{n}\theta^{n}\lambda^{n}\prod_{i=1}^{n}x_i^{\alpha-1}\prod_{i=1}^{n}(1-x^{\alpha})^{\theta-1}\ \prod_{i=1}^{n}\{1-(1-x^{\alpha})^{\theta}\}^{\lambda-1} \qquad (11.1)$$

The log-likelihood function is

$$\ln L(\underline{x}/\lambda)=n\ln\alpha+n\ln\theta+n\ln\lambda+(\alpha-1)\sum_{i=1}^{n}\ln x_i+(\theta-1)\sum_{i=1}^{n}\ln(1-x_i^{\alpha}) + (\lambda-1)\sum_{i=1}^{n}\ln\{1-(1-x_i^{\alpha})^{\theta}\} \qquad (11.2)$$

As shape parameter α and θ are assumed to be known, the ML estimator of shape parameter λ is obtained by solving the

$$\frac{\partial\ln L(\underline{x}/\lambda)}{\partial\lambda}=\frac{n}{\lambda}+\sum_{i=1}^{n}\ln\{1-(1-x_i^{\alpha})^{\theta}\}=0$$

$$\Rightarrow\hat{\lambda}=-\frac{n}{\sum_{i=1}^{n}\ln\{1-(1-x_i^{\alpha})^{\theta}\}} \qquad (11.3)$$

CONCLUSION

We define a three-parameter Exponentiated Minimax distribution (EMD) as a generalization of the Minimax distribution. The resulting model is bounded on a (0, 1) support and its shape of the model could be “constant” or “increasing, decreasing” (depending on the values of the parameters). Minimax distribution, Power distribution and Uniform distribution are found to be sub-models of the proposed model. Some statistical properties of this distribution are discussed and studies. It also estimates the unknown parameter λ of this distribution using the method of maximum likelihood estimation.

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