Small Signal Modelling and Controller Design of Boost Converter using MATLAB

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Designing a controller for pulse width modulation (PWM) power converters is a real challenge owing to nonlinear and time-variant nature of switching power converters. PID controllers based on classical control theory is the simplest controller design approach. Nevertheless, the approach is valid only for linear system and hence the converter has to be linearized. The trial and error method of tuning the controller parameters may not give satisfactory results for advanced converters. Hence a systematic software aided controller design is required. This paper presents a systematic approach for controller design of a dc-dc converter using MATLAB. A simple converter like boost converter is taken as an example to illustrate the approach. Small signal modeling of a boost converter is derived theoretically. This is compared with MATLAB generated small signal model and resultant converter transfer functions. The controller design of the linearised converter is done using MATLAB. Finally, the performance of the controller is verified for line and load variations.

Keywords: DC to DC converter, Small-signal modelling, PID controller design, MATLAB

INTRODUCTION

Designing a controller for switching power converters poses an important challenge to power electronics engineers. The reason being switching converters are non-linear and time variant circuits. Classical control theory is applicable only for linear time invariant systems which can be represented by transfer functions. The time-varying nonlinear power converter must be transformed into linear time invariant before we can analyze it by using classical control theory.

A linear time invariant small signal modeling and analysis approach of switching DC-DC converter, called state space average approach, was proposed (Middlebrook R.D and Cuks, 1976) and widely used for simplifying the analysis and design of switching DC-DC converter. The linear controller design can then be carried out for the linear model.

PID control is a traditional linear control method commonly used in many applications. The PID controller is considered as a popular control feedback used in industrial area due to its feasibility and easy implementation in real application. However finding the optimal control parameters is a difficult task many a times. The trial and error method of tuning the controller parameters may not give satisfactory results for advanced converters. Hence a systematic software aided controller design is required. This paper presents a systematic approach for controller design of a dc-dc converter using MATLAB. A simple converter like boost converter is taken as an example to illustrate the approach. However, the approach is equally valid for any general switching power converter.

Section-II presents the small signal modeling of a boost converter. First the small-signal model is derived theoretically and then using MATLAB.

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Next the converter transfer functions are generated using MATLAB. Section-III presents the controller design of the linearized converter using MATLAB. Finally the performance of the controller is verified for line and load variations.

**MATHEMATICAL MODELING**

Figure 1 shows the circuit diagram of a boost converter.

![Boost Converter Circuit Diagram](image)

**Fig. 1.** Boost converter

The component values and nominal operating conditions are:

- \( L = 120 \mu \text{H} \)
- \( C = 50 \mu \text{F} \)
- \( R = 40 \Omega \)
- Load fluctuations
- Input Voltage=12V +/- Line voltage fluctuations
- Regulated Output Voltage=24V
- Nominal duty ratio: 0.5
- Switching frequency=100kHz

**Assumptions:**

1) Components are ideal and lossless
2) Continuous conduction mode
3) Small Ripple approximation

State equations for each interval of operation:

1) Switch S is ON:

\[
\begin{align*}
L\frac{di}{dt} &= v_{in} \\
C\frac{dv}{dt} &= \frac{-v_{c}}{R}
\end{align*}
\]

2) Switch S is OFF:

\[
\begin{align*}
L\frac{di}{dt} &= v_{in} - v_{c} \\
C\frac{dv}{dt} &= i_L - \frac{v_{c}}{R}
\end{align*}
\]

The steady state values of capacitor voltage and inductor current are found to be:

- \( V_{C} = 24 \text{V} \)
- \( I_{L} = \frac{12}{20} \text{A} = 0.6 \text{A} \)

**A. Averaged Large signal modeling**

**Assumptions:**

1) Natural frequencies are much lower than switching frequency.
2) Duty ratio variation is negligible within one switching time period.

The switching ripples in the inductor current and capacitor voltage waveforms are removed by averaging over one switching period (R. W. Erickson, D. Maksimovic, 2001). The resultant average circuit model is shown in Fig 2.

![Averaged Circuit Model](image)

**Fig. 2.** Nonlinear averaged circuit model of the boost converter.

The resultant averaged state space model for a boost converter is

\[
\begin{bmatrix}
\dot{v}_{c} \\
i_L
\end{bmatrix} =
\begin{bmatrix}
-1 \\
\frac{RC}{L}
\end{bmatrix}
\begin{bmatrix}
i_L \\
v_c
\end{bmatrix} +
\begin{bmatrix}
0 \\
\frac{1}{L}
\end{bmatrix} v_{in}
\]

where \( \bar{v}_c \) and \( \bar{i}_L \) are averaged capacitor voltage and inductor current over one switching time period [3].

The equation (1) results in a non linear model since duty ratio, which is the control input, is multiplied by a state variable in averaged model output.

**Fig. 3.** Boost converter Output: Actual Circuit Vs Averaged model

A comparison of average model output with actual converter output is shown in Fig 3. The waveforms look exactly similar except for the fact that switching ripple is completely absent.
B. SMALL SIGNAL-LINEARISED AVERAGE MODEL

Linearised models are the starting point for controller design and stability analysis. Upon averaging, though the switching harmonics have been removed, the resultant average model is still non-linear (D. Maksimovic, 2001). Hence it has to be linearised using small-signal linearisation technique. The steps to follow to derive the small signal linear model are outlined below (A. Kislovski and R. Redl, 1991).

1) Define state variables and input variables.
2) Identify the operating point around which linearization has to be carried out.
3) Introduce perturbations (small ac variations) in input variables. This results in perturbations in all system variables. 4) Apply the average model. Equate the corresponding dc components and variation components separately.
5) Prepare small signal state space model considering only variation components.

Assumptions:
1) Perturbations are smaller than their steady state values.

Notations used:
Capital Letters: Steady state values
Small letters: Instantaneous values
Capped letters(x): Perturbations around steady state values
Dashed letters(d‘): Compliment or (1-d)
Bar on top(x): Averaged quantities

The input quantities to the converter are input voltage \(v_{in}\) and control input duty ratio \(d(t)\). To construct a small-signal model at the steady state operating point, we assume slight perturbations in input voltage and the duty cycle around their steady state value.

\[
\begin{align*}
\text{vin}(t) &= V_{in} + \nu^*\text{vin}(t) \\
n(t) &= D + \nu^*n(t) \\
i_L(t) &= I_L + \nu^*i_L(t) \\
v_C(t) &= V_C + \nu^*v_C(t)
\end{align*}
\]

Substituting this in the average model yields

\[
\begin{pmatrix}
V_C + \nu^*v_C(t) \\
I_L + \nu^*i_L(t)
\end{pmatrix}
= 
\begin{pmatrix}
\frac{-1}{RC} & \frac{(1-D)\nu^*}{RC} \\
\frac{-1}{L} & \frac{0}{L}
\end{pmatrix}
\begin{pmatrix}
V_C + \nu^*v_C(t) \\
I_L + \nu^*i_L(t)
\end{pmatrix}
\]

Equating the small variations on both sides and neglecting higher order terms yields,

\[
\begin{pmatrix}
\nu_C(t) \\
\nu_i_L(t)
\end{pmatrix}
= 
\begin{pmatrix}
\frac{-1}{RC} & \frac{D\nu^*}{RC} \\
\frac{-1}{L} & \frac{0}{L}
\end{pmatrix}
\begin{pmatrix}
\nu_C(t) \\
\nu_i_L(t)
\end{pmatrix}
+ 
\begin{pmatrix}
\frac{-1}{RC} & \frac{D\nu^*}{RC} \\
\frac{-1}{L} & \frac{0}{L}
\end{pmatrix}
\begin{pmatrix}
\nu^*v_C(t) \\
\nu^*i_L(t)
\end{pmatrix}
\]

C. Linearisation in MATLAB

The linearised small signal model (2) can be computed using MATLAB. The steps to follow are outlined below:

1) Develop the average model for the boost converter in SIMULINK environment.
2) Specify linearisation input and output points. For the present context, input voltage and duty ratio are the input points. Mark the capacitor voltage as output linearization point.
3) Goto Analysis→Control design→Linear analysis in SIMULINK model
4) In Linear analysis tool window, create new operating point using ‘Trim model’. Select the operating point specification as steady state and start trimming. This generates the steady state operating point.
5) Now select the newly generated operating point variable in place of Operating point in Linear analysis tool.
6) Choose bode plot to create a linearised model of the system.

![Fig. 4. Simulation model used for averaged boost converter](image)
D. Converter transfer functions from small signal state space model

Open loop plant transfer function is the primary entity required in classical controller design. Using MATLAB linearisation, the control to output transfer function and input to output transfer function can be obtained as shown in Fig 6. The well-known right half plane (RHP) zero is observed in the converter transfer function. The effects of RHP-zero on controller performance is explained in (V. Michal, D. Cottin, P. Arno, 2016).

The control (Duty ratio) to output (Vc) transfer function is the important quantity for controller design and is analyzed further in this paper. The bode plot of control to output transfer function is shown in Fig 7. The gain crossover frequency is observed to be greater than phase crossover frequency which implies the corresponding closed loop system is unstable.

**CONTROLLER DESIGN**

The bode plot of control (Duty ratio) to output(Vc) transfer function indicates that the corresponding closed loop system is unstable (Fig 7). Hence the loop transfer function has to be modified by introducing a proper controller. A typical closed loop control system is created where converter transfer function acts as plant transfer function. The MATLAB model illustrating the same is shown in Fig 11.

There are standard procedures available in MATLAB to optimise the control parameters of PID block. However, these optimization procedures are applicable only to linearised plant models. This is the primary reason for linearising the converter model in Section II. PID controller for linearised converter model is optimised...
using PID tuner and the resultant bode plots and step response are shown in Fig 9 and Fig 10. It is clear that the corresponding closed loop system is stable with positive gain margin and phase margin.

**Fig. 8.** Simulation model for controller design

**Fig. 9.** Closed loop response to step change in reference voltage

A. Verification of controller performance

The designed controller has to be verified for variations in input voltage and load current. The range of variations the controller can handle also needs to be examined. The simulation model for controller verification against line and load variations is shown in Fig 11.

1) Variation in input voltage: The typical input voltage is +12 V. Due to input fluctuations, the voltage is varied in the range 18 V to 6V. The output is regulated to 24 V irrespective of the variations in inputs (Fig 12). The regulation action is reasonably fast with minimum oscillations.

**Fig. 10.** Bode plot of small signal boost converter with controller

**Fig. 11.** Simulation model for controller verification

**Fig. 12.** Output voltage regulated to 24V against input voltage variations

2) Variation in Load current: The nominal load current is 0.6A. Due to load fluctuations, the load current is varied in the range 1.2 A to 0.1A. The output is regulated to 24 V irrespective of the variations in load (Fig 12). The regulation action is reasonably fast with minimum oscillations.

**Fig. 13.** Output voltage regulated to 24V against load current variations

**CONCLUSION**

In this paper a systematic approach for PID controller design of power converters using MATLAB is explained. Boost converter is taken as an example to illustrate the approach. The theoretical small-signal modelling is verified with software generated small signal model. This approach can be extended for other advanced power converters where theoretical small-signal modelling and controller design would be tedious. Finally, the designed controller is verified against line and load variations.
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