Energy in form of space may solve the dark energy problem

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A review of recent observations suggests a universe that is light weight (matter density is \(1/3\)rd of the critical value), accelerating and flat. This implies the existence of a cosmic Dark Energy that overcomes the gravitational self-attraction force of matter and causes the accelerating expansion. Finding out the cause of expansion and acceleration of the universe is a challenging job in present day cosmology. Cosmological models with different types of dark energy are becoming viable standard models to analyze and simulate experimental data from a number of high red shift supernovae. In this article, physical significance and analytical expression for dark energy related to total energy (or energy density) and matter (or matter density) in the universe is presented. It is assumed that ‘space’ or ‘vacuum’ is another form of energy (other form is mass which is related as \(E = mc^2\)). With this assumption new cosmological equation of state is constructed which is in very good agreement with present observations. Thus energy evolves from matter to radiation to space. It is also predicted that the existence of a fundamental particle with mass less than the mass of a quark is possible.

**Key words**: Dark Energy, red shift, cosmological constant, accelerating universe, space energy

**INTRODUCTION**

To explain a light weight (Trimble 1987 and Gott *et al* 1974), accelerating (Bahell *et al* 1999, Riess *et al* 1998 and Schmidt *et al* 1998) and flat (Ostriker and Steinhardt 1995, Bernardis *et al* 2000 and Perlmutter *et al* 1999) universe, existence of a cosmic dark energy (Perlmutter *et al* 1999 and Tonry *et al* 2003) is granted by the cosmologists. Several attempts have been done (Seo and Eisenstein 2003, Jain and Bernstein 2004, Blake and Glazebrook 2003 and Mainini *et al* 2003) to explain the nature and behavior of dark energy. The standard Big-Bang model of the expansion of the universe from a hot dense gas left several question unanswered about the formation and structure of the present universe. Einstein’s theory of general relativity fixes the total energy density (including matter and radiation) of the universe precisely equal to the critical value, \(\rho_c = (3H_0^2)/(8\pi G)\) which is nearly \(10^{-29}\)gm cm\(^{-3}\), where \(H_0\) is the current value of the Hubble parameter and \(G\) is Newton’s gravitational constant. Measurement shows that the total mass density identified by observations still fell short of the critical value (Trimble 1987 and Gott *et al* 1974). Thus we need some extra parameter to satisfy Friedmann’s equation (Friedman *et al* 1998) of state which may be given as \(\sum \Omega_i = 1\) where \(\Omega_i\) is given as \((8\pi G \rho_i)/(3H^2)\) where \(H = H(t)\), the present value of Hubble parameter and \(\rho_i\) is the density of \(i^{th}\) component.

Einstein’s equation (Einstein 1917) of state is

\[
3(\dot{a}/a)^2 = 8\pi G \rho + \Lambda c^2 - 3Kc^2/a^2
\]

(1)
and
\[ \dot{a}/a = -4/3 \pi G (\rho + 3p/c^2) + (\Lambda c^2)/3 \]  

(2)

Where ‘\( a \)' is scale factor, \( \Lambda \) is cosmological constant and ‘\( \omega \)' is an arbitrary constant which may be taken as unity for simplicity. \( \rho \) is the pressure and \( K \) is the curvature of the space. Combined measurement of the cosmic microwave background (CMB) temperature fluctuations and the distributions of galaxies on large scale indicate that the universe may be flat (Ostriker and Steinhardt 1995, Bernardis et al 2000 and Perlmutter et al 1999), consistent with the standard inflationary prediction (Guth 1981, Linde 1982 and Albrecht and Steinhardt 1982). Thus consideration of flat universe impose another restriction on Equation (1) as \( K = 0 \). To account a flat universe of low mass density as expected from the inflationary theory, one should consider an additional component in equation of state. \( \Lambda \), known as cosmological constant, introduced in the cosmological equation of state by Einstein for a static universe. In reality, \( \Lambda \) is very important parameter to predict the history and future of the universe. For a negative value of \( \Lambda \), there will be deceleration, for positive value satisfying the condition \( \Lambda = 4\pi G (\rho + 3p) \), universe would be static. Thus, only a sufficient positive large value of \( \Lambda \) supports an accelerating universe. The discovery of cosmic expansion by E. P. Hubble in 1929 negotiate the idea of a static universe and researches in last eight decades involved to examine whether the expansion of the universe is accelerating, decelerating or static. S. Perlmutter et al in 1998 with the help of supernovae-a data analysis conclude that the expansion of the universe is accelerating. From Equation (1) it is clear that for an accelerating universe the unknown component must be greater than \( 4\pi G (\rho + 3p) \). According to general relativity, the dark energy should have negative pressure which rule out the usual suspect for dark energy as in form of cold dark matter (Eisenstein et al 1999), neutrinos, radiation and kinetic energy as they have positive pressure. There are several theories of dark energy such as cosmological constant explanation (Sahani 2002 and Sahani and Starobinsky 2000), evolving scalar field theory (Ratra and Peebles 1998), a frustrated network of topological defects (such as strings or walls) where \( \omega = - n/3 \) (n is the dimension of the defect) (Spergel and Pen 1997), the quintessence theory (Caldwell et al 1998 and Weiss 1987), dynamical dark energy model (Mainini et al 2003). All of these theories explain partly about observed results. But the nature of dark energy is still open for debate.

To address the dark energy problem, energy to space conversion is considered here. It is considered that, there is a finite contribution of space to Friedmann’s equation. Contribution of vacuum energy in Friedmann’s equation is also considered in cosmological constant approach (Sahani and Starobinsky 2000) but in this approach it is considered that vacuum energy is related to the radiation energy through Einstein’s mass-energy relation (Einstein 1917). It explains acceleration problem through space creation from radiation and examine the existence of energy in form of space which certainly discovers the fundamental physics underlying in the dark energy problem. As space is created from radiation, with time, vacuum energy density remains constant while the universe expands in an accelerating way. Thus there is no need for fine tuning of vacuum energy density as it requires for cosmological constant approach (Sahani 2002) of dark energy. Using this approach it is possible to construct cosmological parameters in a general form. Also cosmological equation of state is defined using this approach. The assumption that space is created from radiation explains the accelerating nature of our universe. This model, in few respects, is similar to the dynamical dark energy model (Mainini et al 2003). Theoretical calculation of \( \Omega_k \) (dark energy density parameter) using this method is found to be 0.72 which shows very good agreement with the observed results reported in several research articles (Daly and Djorgovski 2003, Huterer and Turner 2001 and Riess et al 1998). Using some simple analytical methods the expansion and acceleration rate of the universe is calculated on the basis of red-shift (z).

THEORY

According to Big-Bang theory (Joseph Silk 2009 and Simon Singh 2005) the universe was in a very high density state and then expanded (Discovery Science 2014). At the initial state of the universe it was in a state of singularity (according to general relativity). A gravitational singularity or space-time singularity is a location where the quantities that are used to measure the gravitational field become infinite and does not depend on the coordinate system. These quantities are the scalar invariant curvature of space-time which includes a measure of the density of matter. It implies that, at the initial state i.e. for space-time singularity, gravitational field would be infinite if space-time = 0 and matter density = 0. Thus, for any time (\( t > 0 \), space-time > 0. From stress-energy tensor (sometimes termed as stress-energy-momentum tensor or energy-momentum tensor) we can say that space-time > 0 for both space and time are positive or both space and time are negative. At space-time = 0, either t =0, space =0 or both are 0. Thus, there is a chance that space > 0 for both space-time = 0 and space-time > 0. From this observation we can conclude that space may be invariant with time. But, to satisfy the condition that space-time > 0 for time < 0 we should have space < 0. This clearly indicates that space also varies with time. The best we can consider, at time = 0, space = 0. This space is a physical space. If we consider that at singularity point time = 0, space = 0 and energy (or mass) = infinity then the question arises
from where space came from? There is only one possibility that space is created from energy or mass. $T^{00}$ in the stress-energy tensor is energy density. Thus, using stress-energy tensor we can find a

space-energy relation. Physical space or vacuum has definite energy which is known since 1948 when Hendrik B. G. Casimir and Dirk Polder predicted the existence of a tiny attractive force between closely placed metal plates due to resonances in the space between them. This is now known as the casimir effect. It is therefore believed that the vacuum energy is "real" in the same sense that more familiar conceptual objects such as electrons, magnetic fields, etc., are real. Not only that, to be a real vacuum it should be particle free. Then, how energy may exist in absence of particle? Is vacuum or a physical space is one form of energy (or matter)? If so, how is it related to the other two forms?

Let us consider de Broglie wave particle duality relation,

$$p = m v = (h / \lambda)$$

where $p$ is momentum, $m$ is mass, $v$ is velocity, $h$ is plank's constant and $\lambda$ is wavelength. For $m = 0$ or $v = 0$ or both are 0, $\lambda = \infty$. We have considered that for vacuum, $m = 0$. Thus, we can say, vacuum or space is one form of energy with infinite wavelength or 0 momentum. For vacuum or space, $v$ may be 0 i.e. space is static. But, for radiation, $p \neq 0$, hence, radiation contains both mass and velocity. On the other hand, if $m \neq 0$ and $v = 0$, then we have matter only phase. At $t = 0$ i.e. at space-time = 0, it was a matter domain universe and with positive time matter starts to form radiation and space. As space is the upper limit of de Broglie wavelength and matter is the lower limit of it, our universe is moving towards a space domain universe through present state of matter, radiation and space.

At present day cosmology positive vacuum energy is well known though there is a debate regarding its exact value. We know that mass ($m$) of a matter is related to energy ($E$) as

$$E = mc^2$$

where $c$ is the velocity of light. Using Equation (3) we can calculate energy (radiation) equivalent of space. If $V$ is the volume of the space (vacuum) and $\rho$ is the energy density of space then energy density of space would be (considering the relativistic effect of energy density)

$$\rho = E / (V c^2)$$

Thus, volume ($V$) of space (vacuum) created from $E$ amount of energy would be

$$V = E / (\rho c^2)$$

These are different forms of space energy relation. According to this approach, mass equivalent of a physical vacuum is a constant parameter and space is created from radiation without changing the vacuum energy density. A differentiation of Equation (5) with respect to time would give us the space creation rate which directly connected to the rate of expansion of the universe and a second order differentiation would give the rate of acceleration of the same. Thus, the rate of expansion is $1/(\rho c^2)(dE/dt)$. This expression is same as the expression for volume expansion of the universe if we consider it as adiabatic one with a negative pressure and $\omega = -1$. The expansion rate of the universe at a red-shift ($z$) is $1/(\rho c^2)(dE_T/dt)$ where $E_T$ is total radiation energy enclosed within the space defined by the distance related to $z$. The red-shift $z$ due to expansion of the universe is thus,

$$z = D \times (d\rho_E/dt)/(3c\rho_s (\rho_s - \rho_E))$$

where $\rho_E$ and $\rho_s$ are energy density of radiation and space. Hence, scale factor ($a$) is defined as-

$$a = \exp (2\rho_E - \rho_s)/(6\rho_s)$$
Thus, the equation of state is as follows,

\[
\frac{\dot{a}}{a} = \frac{\Omega_m (2\Omega_s - \Omega_m)(\Omega_s + \Omega_m)}{6\Omega_s^2} \times \frac{d\rho_E}{dt}
\]  

(8)

Equation (7) and Equation (8) represent cosmological equation of state using this approach. \( \Omega_m \) and \( \Omega_s \) are density parameters of matter (including Energy in form of radiation) and space. These equations are similar to Einstein’s equation of state for matter and energy with an additional term, the energy diffusion rate \( (d\rho_E/dt) \) which is same as space creation rate \( (dv/dt) \). In this equation, matter density parameter and space density parameter (which may be consider as dark energy parameter) appears in product form which is due to the interactions between them. In this model, Hubble parameter is modified because the radiation energy density \( \rho_E \) is a function of time \( (t) \). As more and more space is created, pressure of a matter decreases and hence rate of evaporation of matter to radiation increases i.e. \( \rho_E \) increases with time. Hence, \( H = H(t) \).

Energy diffusion rate of a radiation may be measured by measuring the change of its wave length \( (\lambda) \). Wave length \( (\lambda) \) of a radiation at any time \( (t) \) may be given as -

\[
\lambda = \lambda_0 \left(1 + \theta t^{1/2}\right)
\]  

(9)

where \( \theta \) is a constant and \( \lambda_0 \) is initial wavelength of that radiation. Hence redshift \( (z) \) of a light emitting source due to diffusion of energy is \( \theta t^{1/2} \). We should add this correction term to the previous equation of \( z \) and get an exact equation for \( z \) as

\[
z = D \times \frac{(d\rho_E dt)/(3\rho_c(\rho_s - \rho_E)) + \theta \times (D/c)^{1/2}}{(3cp_s(\rho_s - \rho_E))}
\]  

(10)

For small value of \( D \), \( z \) is proportional to \( D \) which confirms Hubble’s law (Hubble 1929), but for large value of \( D \), due to the presence of second term non linearity appears which agrees with present observations (Bahell et al 1999).

Another interesting key feature of this approach is that it satisfies a negative bulk pressure condition for the dark energy. The bulk pressure is significantly negative and \( \omega < -1/3 \) where \( \omega = p/\rho \), \( p \) is the pressure and \( \rho \) is the density. The first condition is needed to ensure accelerating expansion and to avoid interference with a long matter dominated era during which structure was formed; the second condition is needed to satisfy that the dark energy escapes detection in gravitationally bound systems such as clusters of galaxies. Less negative pressure is permitted when the fraction of critical density contributed by dark energy is larger. A general equation of state for matter as well as for dark energy may be stated as \( p = \omega \rho c^2 \), \( \omega \) is the parameter mentioned above and \( c \) is r.m.s velocity of the element concerned. For a gas \( \omega = 1/3 \) and \( c \) is the r.m.s. velocity of the gas molecule. In case of a solid material numerical value of \( c \) and \( p \) is nearly 0. Radiation also shows a positive value for \( \omega \). The value of \( \omega \) for a vacuum space is derived analytically in the following way using this approach. We know that, energy \( (E) = \text{force} (F) \times \text{displacement} (S) \). Thus, the rate of change of energy is

\[
\frac{dE}{dt} = p \times \frac{dV}{dt}
\]  

(11)

According to our consideration, energy of radiation decreases with increase of space. Thus, from Equation (5) we get,

\[
\frac{dV}{dt} = -\frac{1}{(\rho c^2)} \times \frac{dE}{dt}
\]  

(12)

hence,

\[
\omega = \frac{p}{(\rho c^2)} = -1
\]  

(13)

These values are similar to recent results reported using the WMAP satellite, also in combination with the 2dF Redshift Survey (Tonry et al 2003). Thus, it may be stated that the so called dark energy is nothing but the space energy described in this article.

RESULTS AND DISCUSSIONS

In theory section, physical space or vacuum is described as one of the boundary condition of de Broglie wave equation. With the help of stress-energy tensor and de Broglie wave particle duality relation between space (vacuum) and radiation energy is derived which is used to construct new equation of state of our universe. Equation (7), Equation (8) and Equation (10) are such equations. To verify the correctness of these equations different cosmological parameters
are calculated using these equations and their variation is studied and compared with experimental results or previous theories.

**Variation of Hubble's constant (H) at different time span:**

![Graph showing variation of Hubble's constant with time](image)

*Figure 2: variation of Hubble constant with time (t)*

The proportionality between recession velocity and distance in the Hubble law is called the Hubble constant or more appropriately the Hubble parameter \( H \) since it does depend upon time. The variation of \( H \) with time is one of the important properties of any cosmological model. A plot of \( H \) vs \( t \) is presented in Figure 2. Brown and sky blue colored lines are corresponding to any cosmological model with a universe of accelerating followed by decelerating expansion. Green and blue colored lines are corresponding to this model with accelerating universe for two different \( \theta \) in Equation (9). For this model \( H(t) \) is nearly constant for small value of \( t \) but increases exponentially with large value of \( t \) which implies the accelerating expansion of the universe.

**Variation of optical magnitude with redshift:**

![Graph showing variation of apparent magnitude with log of redshift](image)

*Figure 3. Change of apparent magnitude with log of redshift (log Z)*

Change of apparent magnitude \( m \) of a cosmological object with \( \log Z \) is presented in Figure 3. Red line is corresponding to this model and green line is corresponding to Hubble's law. Blue line is for experimental values taken from literature. From this plot it is observed that for small value of \( \log Z \), Hubble's model is very good. But for high redshift, space energy model is better compare to experimental results.

**Variation of cosmological distance \((D)\) and luminosity distance \((D_l)\) with redshift:**

Luminosity distance \( D_l \) is defined in terms of the relationship between the absolute magnitude \( M \) and apparent magnitude \( m \) of an astronomical object as follows

\[
D_l = 10^{\frac{(m-M)/5 + 1}{}}
\]  

(14)

Variation of astronomical distance \( D \) and luminosity distance \( D_l \) with redshift \( Z \) is presented in Figure 4 using Equation 10 and Equation (14). Red line is for \( D \) and green line is for \( D_l \). From the plot we observe that for small value of \( D, Z \) is proportional to \( D \) which satisfies Hubble's law. For large value of \( D \), due to the presence of the second term in Equation 10, non-linearly appears which agrees with present observations (Bahell et al 1999). This agreement clearly
shows the correctness and effectiveness of this approach.

CONCLUSIONS

Using this approach, generation, evolution and nature of dark energy may be realized through very simple mathematics. It is possible to derive important cosmological parameters using this method directly from some measurable quantities. Without mathematical support there is another proof of this model. According to Big-Bang model at the initial stage of the universe temperature of the universe was very high and hence the radiation was in the region of γ-ray only. But at present age the higher energy density of the universe lies at microwave region according to the cosmic microwave background (CMB) anisotropy detected by the COBE satellite in 1992 followed by a series of ground and balloon based experiments. Thus the expansion of the universe may be considered as adiabatic expansion where space is created on expenditure of energy. We also conclude that there are three different forms of energy in this universe as matter, radiation and space. Thus, our universe starts from a matter only stage where there was only mass with infinite density but no space and radiation. After Big-Bang, mass starts to convert to form energy and then energy to space. Thus, an object with very high gravitational force, say black hole, can squeeze the space to form energy out of it and make another Big-Bang. In this sense energy, matter and space may evolve in a cyclic path. There is another interesting property of dark energy under this model coming from the ω equation (Equation 13) that the so called vacuum may contains particles of very little mass (may be less than the mass of a lepton) when λ→ 0, which moves with a velocity higher than the velocity of light due to which its pressure is negative. These fundamental particles may be smaller than quarks and leptons. These unknown particles should be highly anti-symmetric or they may be treated as anti-gravity particles.

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