



Research Note

Some Notes on Self-similar Axisymmetric Force-free Magnetic Fields and Rotating Magnetospheres

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An axisymmetric force-free magnetic field in spherical coordinates has a relationship between its azimuthal component to its poloidal flux-function. A power law dependence for the connection admits separable field solutions but poses a nonlinear eigenvalue boundary-value problem for the separation parameter (Low and Lou, *Astrophys. J.* 352, 343 (1990)). When the atmosphere of a star is rotating the problem complexity increases. These Notes consider the nonlinear eigenvalue spectrum, providing an understanding of the eigen functions and relationship between the field's degree of multi-polarity, the rotation and rate of radial decay as illustrated through a polytropic equation of state. The Notes are restricted to uniform rotation and to axisymmetric fields. Dominant effects are presented of rotation in changing the spatial patterns of the magnetic field from those without rotation. For differential rotation and non-axisymmetric force-free fields there may be field solutions of even richer topological structure but the governing equations have remained intractable to date. Perhaps the methods and discussion given here for the uniformly rotating situation indicate a possible procedure for such problems that need to be solved urgently for a more complete understanding of force-free magnetic fields in stellar atmospheres.

Key words: Rotating stellar atmospheres, axisymmetric magnetic fields, polytropes, nonlinear, eigenvalue equations

INTRODUCTION

In discussions of the behavior of force-free magnetic fields in stellar atmospheres, as perhaps best exemplified by the pioneering work of Low and Lou (1991) for the solar atmosphere and many later publications by a variety of authors, the main theme has been, and continues to be, the ability to solve the relevant nonlinear equations that arise expressing the polytropic magnetic entropy as a function of the vector potential. The fundamental equations continue to be challenging and have so far defied general solution although particular aspects are now relatively well understood. For example situations where one focuses solely on axisymmetric fields together with the demand of self-similarity have led to a remarkable richness of numerical solution behaviors all indicating the complexity of dipolar and multipolar behaviors of such fields. Indeed, the graphic realizations of such numerical solutions show strong visual resemblances to magnetic fields in the solar corona, perhaps so indicating one is starting to obtain a good understanding of such force-free fields. The corresponding non-axisymmetric problems remain formidable concerns to this day.

Numerical models of force-free fields in realistic 3D geometry are a standard tool in solar physics, used to infer coronal field structures from polarimetric detection of the fields in the much denser photosphere below the corona (Aly and Amari, 2007; Low and Flyer, 2007; Wiegmann, 2008). Low and Lou (1990) presented a class of axisymmetric force-free fields in spherical coordinates that has become widely used in this area of work. In general the paucity of explicit solutions to the force-free equations means that the Low and Lou (1990) fields are useful as simple solutions illustrating possible coronal field configurations in a Cartesian model.

Locating the origin of the field's spherical coordinates below a plane representing the base of the corona, the coronal field modeled in the unbounded space above has a geometrically realistic appearance in which the hidden intrinsic axisymmetry may be regarded as incidental. These solutions have become standard benchmarks for evaluating the veracity of 3D numerical codes developed to construct general force-free fields in the corona (Amari and Aly, 2010; Wiegmann and Inhester, 2010; Judge et al., 2010; Fuhrmann et al., 2011; Aschwanden et al., 2012; Malanushenko et al., 2012; Guo et al., 2012; Fan et al., 2012; Jiang et al., 2012; Yan et al., 2013; Wang, Yan, and Tan, 2013; Contopoulos, 2013; Aschwanden, and Malanushenko, 2013; Amari et al., 2013; Gilchrist, and Wheatland, 2014; Prasad, Mangalam and Ravindra, 2014; Tadesse et al., 2014). These solutions have also found other astrophysical applications examples are in studies of rotational winding of magnetic fields, accumulation of magnetic helicity, neutron star magnetospheres, and star-planet magnetic interactions (Wang and Lou, 2014).

A major underlying assumption for the progress made to date has been the requirement that the stellar force-free atmosphere be static. This assumption has enabled one to ignore centrifugal forces that would attempt to cause fields to expand away from the stellar surface. The purpose of this short Note is to include such effects for an atmosphere in constant (non-differential) rotation so that one has a model that is likely somewhat closer to reality than a static atmosphere provides. While it is presumably possible to handle non-axisymmetric fields under such rotating atmosphere conditions that set of nonlinear magnetic field problems is even more forbidding than the situation without rotation.

In order to make some progress with rotating atmospheres containing equilibrium force-free magnetic fields attention is restricted here to axisymmetric fields in an atmosphere undergoing constant rotation and to self-similar behaviors that seem the most tractable of the many possible nonlinear problems.

Quantitative Development

Consider the equilibrium equations describing the steady dynamical structure of a gas of pressure p and density ρ undergoing rotation at an angular speed ω in the presence of a magnetic field \mathbf{B} .

Using spherical coordinates (r, ϑ, φ) and setting $R = r \sin \vartheta$, which is just the cylindrical distance from the axis of rotation, one has the usual steady-state force balance described through

$$(1/4\pi)(\nabla \times \mathbf{B}) \times \mathbf{B} - \nabla p + \rho \mathbf{R} \omega^2 = 0 \tag{1}$$

Equally, one has the requirements

$$\nabla \cdot \mathbf{B} = 0 \tag{2}$$

together with the polytropic equation of state

$$p = F \rho^\gamma \tag{3}$$

Here γ is the polytropic index and F is considered to be the polytropic entropy for each magnetic flux tube so that F may vary from tube to tube. The vector \mathbf{R} is just the usual cylindrical radial vector.

This general set of equations is now reduced to the situation of axisymmetry solely and then two special cases are given to illustrate the patterns of behavior that one can obtain.

An axisymmetric magnetic field is given through

$$\mathbf{R} \mathbf{B} = (r^{-1} \partial A / \partial \vartheta, -\partial A / \partial r, Q) \tag{4}$$

Because the situation is axisymmetric there is no force allowed in the φ direction so that equation (1) then requires Q be a function solely of A .

Using equation (4) in equation (1) for force balance in the radial (r) and azimuthal (ϑ) directions, respectively, one has

$$(1/(4\pi R^2))(LA + QdQ/dA)\nabla A + \nabla p + \rho \mathbf{R} \omega^2 = 0 \tag{5}$$

with
$$L = \partial^2 / \partial r^2 + (1 - \mu^2) r^{-2} \partial^2 / \partial \mu^2 ; \mu = \cos \vartheta; R = r \mu \tag{6}$$

The pressure, $p(r, \vartheta)$ can be expressed equally as $p(R, A)$ with no loss of generality so that with

$$\nabla p = \mathbf{R} / R \partial p / \partial R + \nabla A \partial p / \partial A \tag{7}$$

one can write the radial and azimuthal force components as

$$LA + QdQ/dA + 4\pi R^2 \partial p / \partial A = 0 \tag{8}$$

$$\partial p / \partial R = \rho R \omega^2 \tag{9}$$

For the polytrope (equation 3) one then has

$$\rho = G(A)^n (\omega^2 R^2 / 2)^n \tag{10}$$

and

$$p = F(A) G(A)^{n+1} (\omega^2 R^2 / 2)^{n+1} = P(A) (\omega^2 R^2 / 2)^{n+1} \tag{11}$$

where $G(A) = ((n+1)F(A))^{-1}$ and $\gamma = 1 + 1/n$.

Equation (8) then reduces to a PDE for A as:

$$LA + QdQ/dA + 4\pi r^2 \sin^2 \vartheta dP(A)/dA (\omega^2 R^2 / 2)^{n+1} = 0 \tag{12}$$

which is to be solved for given $P(A)$ and $Q(A)$ - the profiles of the polytropic entropy and the azimuthal field across the constant- A field lines of the magnetic field projected on the r - ϑ plane.

Self-Similar Fields

Separable solutions exist to equation (12) when one writes

$$A = \Phi(\mu)/r^a \tag{13}$$

together with

$$QdQ/dA = q_0^2 A^N \tag{14a}$$

and

$$dP(A)/dA = p_0 A^M \tag{14b}$$

where a, N, M, q₀ and p₀ are constants.

Separability then demands the relations between a, N and M be

$$N = (a+2)/a ; M = (a+6+2n)/a \tag{15}$$

and one then has the ordinary differential equation to solve

$$(1-\mu^2)d^2\Phi/d\mu^2 + a(a+1)\Phi + q_0^2\Phi^N + \Lambda(1-\mu^2)^{2+n}\Phi^M = 0 \tag{16}$$

subject to the boundary conditions

$$\Phi = 0 \text{ on } \mu = \pm 1 \tag{17}$$

with the constant $\Lambda = 4\pi p_0(\omega^2/2)^{n+1}$.

To illuminate some of the richness of solution behaviors to equation (16) consider first the illustrative situation where q₀ = 0 so that attention is focused on the rotational aspects alone and the corresponding field is purely poloidal. (The second illustrative example reverses this focus by ignoring rotation and concentrating on the potential field behavior so that one can then see the difference in behavior due to the dominance of rotation over the potential aspects- see later for details). Equation (16) then reduces to

$$(1-\mu^2)d^2\Phi/d\mu^2 + a(a+1)\Phi + \Lambda(1-\mu^2)^{2+n}\Phi^M = 0 \tag{18}$$

Inspection of equation (17) shows that there is a close interplay between the value of Λ (the eigenvalue) and the value of Φ on $\mu = 0$ because if one were to rescale Φ by writing

$\Phi = b\Psi$ with $\Lambda = b^{1-M}$ then one obtains

$$(1-\mu^2)d^2\Psi/d\mu^2 + a(a+1)\Psi + (1-\mu^2)^{2+n}\Psi^M = 0 \tag{19}$$

Thus one can either specify the value of Φ on $\mu = 0$ and then use the rescaling to obtain the corresponding value for Ψ on $\mu = 0$ or one can choose a value for Λ and then determine the value of Φ on $\mu = 0$ after one has solved equation (20) using a value for Ψ on $\mu = 0$. Symmetry of the equations then shows that one need only consider solutions even in μ because solutions that are odd in μ manifestly have $\Psi = 0$ on $\mu = 0$ and so do not permit an eigenvalue type of determination for Λ . Note that if one were to choose a value of Ψ smaller than unity at $\mu = 0$ then the nonlinear term will in general be much smaller than the term linear in Ψ (because $M > 1$ for all n and a positive which are the only physically acceptable domains for these two parameters) so that one has then basically a Legendre equation, whose solution is well known. Thus in order to obtain a significant influence of the rotational term on the structure of the magnetic field the choice of Ψ (or Φ) on $\mu = 0$ must be larger than about unity. In the illustrative numerical case given in Figure 1 the choice is $\Phi = 10$. (In Appendix A some general concerns for equation (16) are addressed that prove inordinately useful in guiding thought about how to proceed with various aspects including the proof that the choice of Ψ (or Φ) on $\mu = 0 =$ must be large enough to overcome the negative second derivative effects).

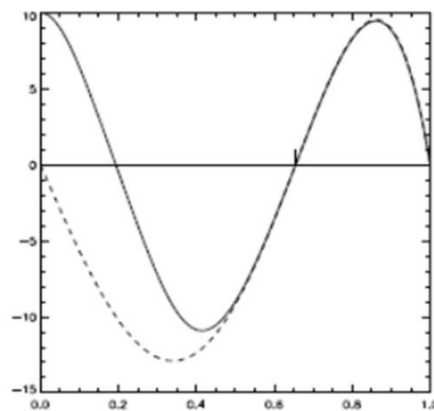


Figure 1. The first eigenfunction $\Phi_1(\mu)$ with $\Lambda = 0.6405$ (solid curve) plotted against μ as abscissa. The dashed curve is the potential solution $\Phi_{pot} = \mu(1-\mu^2)(7\mu^2-3)$ scaled to have the same gradient at $\mu = 1$ as $\Phi_1(\mu)$. Note that the potential solution is asymmetric in μ while the eigenfunction is symmetric.

For the first illustration the choices made are a polytrope with $\gamma = 2$ and so $n = 1$ together with $a = 4$ so that $N = 3/2$ and $M = 3$. These choices are not completely arbitrary because the values chosen for n and a allow field patterns to be more readily sketched than, say, a choice of $\gamma = 5/3$ that would resemble a hydrogen atmosphere while $a = 4$ shows the spatial decay of the magnetic field to be around r^{-6} that would then be limited to the near surface of the atmosphere without the influence of rotation and so one can study better the extensive effects of rotation. To be sure, other values can also be used for the parameters but the pattern of the spatial structures does not vary that much and is well served by the parameters chosen. For $M = 3$ one has $\Lambda = b^{-2}$ so that $\Lambda^{1/2}\Phi = \Psi$. Note then that a “shooting” procedure with ever increasing values for Λ (for $\Phi = 10$ on $\mu = 0$) and using a Runge-Kutta fourth order solver for the basic equation (19) will eventually allow an initial ($\mu = 0$) value of Ψ to be reached such that on $\mu = 1$ the value of Ψ is zero. The resulting eigenvalue is $\Lambda = 0.6405$ and the initial value of Ψ is 8.003. Higher eigenvalues can also be computed as has been shown by Low (2014).

In figure 2 are plotted the field lines of constant flux function A for the first eigenvalue where $A = \Phi_1(\mu)/r^4$ and where the index 1 refers to Φ for the first eigenvalue over the range $1 < r < 3$ where the field is anchored at the atmospheric base $r = 1$. There are 5 sectorial domains between $-1 < \mu < 1$ with only the sectors between $0 < \mu < 1$ plotted due to symmetry.

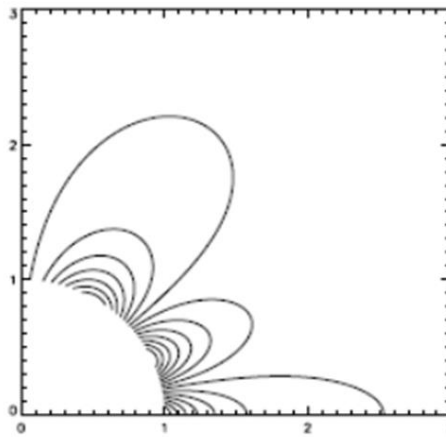


Figure 2. The field lines of constant flux function $A = \Phi_1(\mu)/r^4$ associated with the first eigenvalue $\Lambda = 0.6405$ plotted in the domain $1 < r < 3$ and $0 < \theta < \pi/2$ with symmetry about the equator.

Leave this illustration for the moment and concentrate now on an illustration where the effects of atmospheric rotation are ignorable. The point here is to show as starkly as possible the high degree of contrast between the two illustrations. Thus one now sets $\Lambda = 0$ when the scaling transformation of the first illustration cannot be used.

The general equation (16) can then be written

$$(1 - \mu^2)d^2\Phi/d\mu^2 + a(a+1)\Phi + q_0^2\Phi^N = 0 \tag{20}$$

Note from equation (20) that for $N \neq 1$ one can transform q_0 out of the equation by setting

$$(q_0)^{2/(N-1)}\Phi = U \tag{21}$$

when the equation for U is then

$$(1 - \mu^2)d^2U/d\mu^2 + a(a+1)U + U^N = 0 \tag{22}$$

Note that equation (22) has the same basic structure as equation (19) for the first illustration and so can be solved again with a shooting method for the nonlinear eigenvalue q_0 using the same Runge-Kutta equation solver. However when $q_0 = 0$ such a transformation is no longer possible but then one has the linear equation

$$(1 - \mu^2)d^2\Phi/d\mu^2 + a(a+1)\Phi = 0 \tag{23}$$

representing a potential field. The major point of difference between equations (22) and (23) is that the presence of the term U^N in equation (22) makes the second derivative more sharply negative than the corresponding value in the absence of the term. Thus equation (23) represents the situation with the smallest negative curvature effect of the second derivative *mutatis mutandis* and so provides the greatest contrast to the results including rotation of the first illustration. Accordingly, it is of interest to consider that situation as the ultimate contrasting pattern to the first illustration.

The solution to equation (23) satisfying the boundary conditions for $a = 4$ (a spatial decay given through $A_{\text{pot}} = \Phi_{\text{pot}}(\mu)/r^4$) is

$$\Phi_{\text{pot}} = \mu (1 - \mu^2) (7\mu^2 - 3) \tag{24}$$

This potential field is an odd function of μ with four lobes as shown in Figure 1 by the dashed curve.

Now use the first eigenfunction with rotation $\Phi_1(\mu)$ belonging to the eigenvalue $\Lambda_1 = 0.6405$ to define the normal component of a potential field $A_{\text{pot},1}$ on $r=1$. This normal field component can be written as a linear sum of

$$A_i = \sin\vartheta P_i^1(\sin\vartheta)/r^4, \quad i = 1, 2, 3, 4, \dots \quad (25)$$

Such a field, if set spinning at a rate ω , will throw out some mass to infinity with the field opening and then reconnecting to retain just enough mass to be held by the magnetic field against the centrifugal force. The first illustration is just such an end-member case assuming that the field is rigidly anchored at $r=1$.

Now reconsider equation (1). Take the scalar product with \mathbf{B} to obtain

$$\mathbf{B} \cdot \mathbf{R} \left(\frac{\partial P(R,A)}{\partial R} - \rho R \omega^2 \right) = 0 \quad (26)$$

which is just the statement of hydrostatic balance against centrifugal force along the field noting that A is a constant for each field line. For the $n = 1$ polytrope (i.e. $\gamma = 2$) one has the pressure as

$$p = P(A) (0.5 \omega^2 R^2 \sin^2 \vartheta)^2 \quad (27)$$

decreasing inwards as R^4 from a maximum value at the outermost top radially of each field line. Thus, although the pressure and density decrease outwards as functions of r along radial lines they both increase outwards along individual field lines of constant A so that the hydrostatic pressure gradient is just balancing the centrifugal force because there is no Lorentz force along the field lines. This situation is shown in Figure 3.

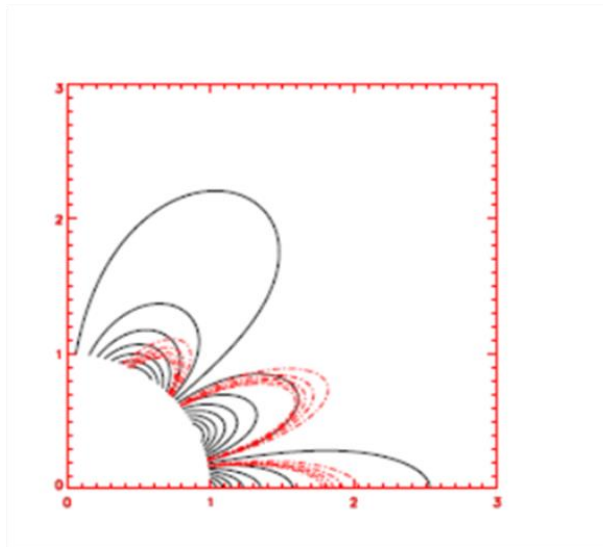


Figure 3. The field lines of constant flux function $A = \Phi_1(\mu)/r^4$ associated with the first eigenvalue $\Lambda = 0.6405$ plotted in the domain $1 < r < 3$ and $0 < \vartheta < \pi/2$ with symmetry about the equator (solid black lines); superposed as broken red lines are curves of constant pressure p . By following an individual field-line of constant A one notes that p is a maximum at a point tangent to a $r = \text{constant}$ and decreases in either direction along the field line to the coordinate origin at $r=1$.

DISCUSSION AND CONCLUSION

One of the major motivations for considering coronal force-free magnetic field behavior in rotating atmospheres is related to the fact that most stars, as far as can be discerned observationally, do rotate. These Notes show the influence of such rotation on the spatial structure of axisymmetric fields, at least in so far as the fields are describable by self-similar behavior. The Notes treat the simplest of all such situations, namely a uniformly rotating atmosphere. Accordingly, effects due to differentially shearing rotational motion are not included nor are effects if the magnetic field is not axisymmetric.

The high degree of nonlinearity even in the simple case discussed here and the determination of the steady-state eigenvalues and eigenfunctions of the nonlinear problem would both indicate that the more general problems, including differential shear and non-axisymmetry, are even more complex. Nor is it clear that such more general situations even have steady-state solutions that are self-similar or have any steady-state solutions at all. The points discussed here would indicate that rotation plays a significant role in changing the force-free magnetic field structure from that which obtains when rotation is ignored. In particular the self-similar requirements of integer parameter values if one is to obtain a solution, and the corresponding multi-pole structure of the field would seem to be stringent requirements. Whether similar requirements are in force in more general situations is unknown.

Perhaps of greatest interest for the solutions obtained here is the behavior of the magnetic field and corresponding gas pressure showing that along field lines the gas pressure decreases inwards while when followed radially the gas pressure decreases outwards- both being consequences of the centrifugal force attempting to eject material.

While the work reported here is limited by the basic requirements of constant rotation and self-similarity nevertheless the pattern of results indicates that the more general problems of such force-free magnetic fields in differentially rotating atmospheres most likely also have unforeseen consequences. Such problems are well beyond the scope of the present work but are currently under investigation. To what degree one will be successful in extracting steady-state patterns of behavior for such complex situations is unclear but, despite the mathematical challenges these problems pose, their investigation is needed if one is to obtain a deeper understanding of force-free magnetic field behavior as it pertains to more applied situations of rotating stars.

The Notes here suggest how one should go about such investigations and the results obtained are sufficiently intriguing as to provide the needed motivation.

Appendix A

Equation (16) is just one member of a class of equations that require special consideration when subjected to the boundary conditions $\Phi = 0$ on $\mu = \pm 1$.

Consider the general member of the class written in the form

$$(1 - \mu^2)d^2\Phi/d\mu^2 + G(\Phi, \mu) = 0 \quad (A1)$$

Where $G(\Phi, \mu)$ is an arbitrary function of its arguments. Rewrite equation (A1) in the form

$$d((1 - \mu^2)d\Phi/d\mu + 2\mu\Phi) - 2\Phi + G(\Phi, \mu) = 0 \quad (A2)$$

Integrate equation (A2) across the domain $-1 < \mu < 1$ using the boundary conditions when it follows that

$$\int_{-1}^1 (G(\Phi, \mu) - 2\Phi) d\mu = 0 \quad (A3)$$

If Φ is of one sign (positive or negative) throughout the domain then so too must be the integral of G . This theorem, when applied to equation (16) where

$$G = a(a+1)\Phi + q_0^2\Phi^N + \Lambda(1 - \mu^2)^{2+n}\Phi^M \quad (A4)$$

implies that

$$\int_{-1}^1 (a - 1)(a + 2)\Phi d\mu + \int_{-1}^1 (q_0^2\Phi^N + \Lambda(1 - \mu^2)^{(2+n)}\Phi^M) d\mu = 0 \quad (A5)$$

Note that if $a > 1$ (or $a = 1$) then there are no solutions satisfying the boundary conditions for which $\Phi > 0$ everywhere (at least this is so for q_0^2 and Λ both positive which are the only allowed physical values). Thus Φ must then change sign at least once in the domain $-1 < \mu < 1$. If there is a change in the sign of Φ then one requires both N and M to be integers. Then one can write $a = 2/(N-1)$ and $n = (M+2-3N)/(N-1)$. Because one is dealing with magnetic fields that decay radially away from the $r=1$ anchoring surface one then requires first that $N > 1$ in order that $a > 0$ and, because the polytrope index $\gamma = 1 + 1/n$, one also requires $M > 3N - 2$.

An alternative is to change the structure of the dependences expressed in equations (14a) and (14b) by replacing A on the right hand side of each equation by $|A|$ so that one does not then run into the problem when Φ can become negative. However this extension would take the problem further afield than is needed in these Notes and so is not considered further here although an investigation is currently underway to figure out the physics and also the difference such changes would produce in the self-similar patterns.

It is in this sense that one can use the general requirements spelled out in this Appendix to guide thought concerning the structure of allowed behaviors for the solutions to equation (16). A similar, but less general, argument for non-rotating atmospheres has already been given in Lerche and Low (2014).

ACKNOWLEDGEMENTS

Much of the work reported here originated with B.C. Low. However his commitment to other projects prior to his imminent retirement ended up with his withdrawal from the paper as co-author- a less than happy circumstance in my opinion. It would have most fitting if it could have been otherwise.

REFERENCES

- Aly JJ, Amari T (2007) Structure and evolution of the solar coronal magnetic field, *Geophys. Astrophys. Fluid Dyn.* 101, 249.
 Amari T, Aly JJ (2010). Observational constraints on well-posed reconstruction methods and the optimization-Grad-Rubin method, *Astron. Astrophys.* 522, id A52.
 Amari T, Aly JJ, Canou A, Mikic Z (2013). Reconstruction of the solar coronal magnetic field in spherical geometry,

- Astron. Astrophys. 553, id.A43.
- Aschwanden MJ, Malanushenko A (2013). Aschwanden, M. J. and Malanushenko, A.,(2013), Solar Phys. 287, 345., Solar Phys. 287, 345.
- Aschwanden MJ, Wuelser JP, Nitta NV, Lemen JR, DeRosa ML, Malanushenko A (2012). First three-dimensional reconstructions of coronal loops with the stereo A+B spacecraft. IV: magnetic modeling with twisted force-free fields, Astrophys. J. 756, id. 124
- Contopoulos I (2013). The state of nonlinear force-free magnetic field extrapolation, Solar Phys. 282, 419.
- Fan YL, Wang HN, He H, Zhu XS (2012). Application of a data-driven simulation method to the reconstruction of the coronal magnetic field, Res. Astron. Astrophys. 12, 563.
- Fuhrmann M, Seehafer N, Valori G, Wiegelmann T (2011). Accuracy of magnetic energy computations, Astron. Astrophys.526, id A70.
- Gilchrist SA, Wheatland MS (2014). Nonlinear force-free modeling of the corona in spherical coordinates, Solar Phys. 289, 1153-1171.
- Guo Y, Ding MD, Liu Y, Sun XD, DeRosa ML, Wiegelmann T (2012). Astrophys.J. 26 760, id. 47.
- Jiang C, Feng X, Wu ST, Hu, Q (2012). Study of the three-dimensional coronal magnetic field of active region 11117 around the time of a confined flare using a data-driven CESE-MHD model, Astrophys. J. 759, id. 85.
- Judge PG, Tritschler A, Uitenbroek H, Reardon K, Cauzzi G de Wijn A (2010). Fabry-Perot versus slit spectropolarimetry of pores and active network. Analysis of IBIS and Hinode data, Astrophys. J. 710, 1497.
- Lerche I, Low BC (2014). A Nonlinear Eigenvalue Problem for Self-similar Spherical Force-free Magnetic Fields, Physics of Plasmas (accepted,in press).
- Low BC, Flyer N (2007). The topological nature of boundary value problems for force-free magnetic field, Astrophys. J. 668, 557.
- Low BC, Lou YQ (1990). Modelling solar force-free magnetic fields, Astrophys. J. 352, 343.
- Low BC (2014), Individual communication.
- Malanushenko A, Schrijver CJ, DeRosa ML, Wheatl MS, Gilchrist SA (2012). Guiding Nonlinear Force-Free Modeling Using Coronal Observations: First Results using a Quasi Grad-Rubin Scheme, Astrophys. J. 756, id. 153.
- Prasad A, Mangalam A, Ravindra B (2014). Separable solutions of force-free spheres and applications to solar active regions, Astrophys. J. 786, id. 81.
- Tadesse T, Wiegelmann T, Gosain S, MacNeice P, Pevtsov AA (2014). First use of synoptic vector magnetograms for global nonlinear, force-free coronal magnetic field models, Astron. Astrophys.562, id.A105.
- Wang L, Lou YQ (2014). Steady-state axisymmetric nonlinear magnetohydrodynamic solutions with various boundary conditions MNRAS 439, 2323.
- Wang R, Yan Y, Tan B (2013). Three-Dimensional Nonlinear Force-Free Field Reconstruction of Solar Active Region 11158 by Direct Boundary Integral Equation, Solar Phys. 288, 507
- Wiegelmann T, Inhester B (2010). How to deal with measurement errors and lacking data in nonlinear force-free coronal magnetic field modelling?, Astron. Astrophys. 516, id A107.
- Wiegelmann TJ (2008). Nonlinear force-free modeling of the solar coronal magnetic field,Geophys. R. 113, A03S02.
- Yan S, Bluchner JJ, Santos C, Zhang H (2013). Evolution of relative magnetic helicity: method of computation and application to a simulated solar corona above an Active Region, Solar Phys. 283, 369.

Accepted 25 September, 2014.

Citation: Lerche I (2014). Some Notes on Self-similar Axisymmetric Force-free Magnetic Fields and Rotating Magnetospheres. Journal of Physics and Astronomy Research 1(2): 007-012.



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