Mechanical wave descriptions for planets and asteroid fields: kinematic model for tidal wave at Earth

Lena J-T Strömberg

Previously Department of Solid Mechanics, Royal Institute of Technology, KTH, Sweden
e-mail: lena_str@hotmail.com

Models with wave dynamics and oscillations in the solar system are presented. A soliton solution (Korteweg-de Vries), for a density field, is related to the formations of planets. A new nonlinear equation for a soliton, will be derived, and denoted ‘J-T equation’. The linearized version has solutions, which are small vibrations with eigen frequency proportional to the parameters describing the soliton wave, around a constant level, which is 2/3 of the maximum soliton density. The location and orbital motion of Mercury and Venus are compared with wave dynamics. The tidal effect for Earth is analysed in terms of dynamics. Related phenomena for other planetary objects are discussed in conjunction with assuming a Roche limit.

Keywords: SOLITON, Korteweg-de Vries, wave velocity, J-T equation, oscillation, Tidal effects, Roche limit

INTRODUCTION

With phenomena in the Solar system as the point of departure, models with wave motions are derived.

Before the formation of planets, it is possible that the solar system was a rotating continuum, and could be described with a density field and continuum mechanics. Here, soliton solutions to a density field are discussed, and a correspondence to the formations of planets is suggested.

With wave dynamics, an analogy with a water wave Falkemo (1980), is considered, such that the ratio of angular velocities for Mercury and Venus can be derived.

In Strömberg (2014), it was shown how the motion in a noncircular orbit could be related to a frequency. Here, these results are exploited for planets subjected to tidal effects, and at the Roche limit. It is shown, that the tidal wave at Earth can be described with the model in Strömberg (2014), applied to sidereal rotation. Some related concepts will also be discussed, for various objects, e.g. fluid satellites and rocks.

SOLITON WAVE

Here, a planet at formation, is considered a densification, such that it relates to a soliton solution of a density field (distribution). A soliton is a solution to Korteweg-de Vries equation. Next, a continuum with density $\rho$, and velocity $u$ will be analysed.

Continuity of mass for a fluid reads $\rho, t + \text{div}(\rho u) = 0$ \hspace{1cm} (1)
**Definition:** A fluid is assumed to move in a rotational motion $\mathbf{u}=\omega \mathbf{r}$, where $\omega$ is constant angular velocity, in a circular orbit with radius $r$.

**Theorem:** The density field at rotational motion have a wave solution, when $\mathbf{u}$ is parallel with $x$, such that $\rho = \rho_0 (x - vt)$, where $v = \omega r$.

**Proof:** Insertion in equation (1).

**Corollary.** One possible wave solution is a soliton
\[ \rho = \rho_0 \text{sech}^2 \left( \frac{(x - vt)}{L} \right) \tag{2} \]
where $L$ is a constant length.

A soliton (in a continuum flow) may be interpreted as a densification, a pre-state to a planet, a larger asteroid in the asteroid belt e.g. Ceres, or a shepherd moon in the planetary rings of Saturn; Pan in the Encke Gap, and Daphnis in Keeler Gap.

A soliton, (which emanates from the solution of Korteweg-de Vries equation), also fulfil other nonlinear differential equations

**Theorem.** The soliton $\rho = \rho_0 \text{sech}^2 \left( \frac{(x - vt)}{L} \right)$ fulfil the nonlinear equation $\rho_{axx} - 4\rho + 6\rho^2/\rho_0 = 0$ \tag{3}
where $\rho_0$ is a constant density, and $a = (x - vt)/L$.

**Proof.** Differentiation and evaluation.

Subsequently, (3) will be denoted the J-T equation for a soliton densification.

A linearization of the J-T equation at constant $x$, gives the equation of a harmonic oscillator. The solution is a small vibration around a constant density.

**Theorem.** Consider solutions to (3), of the kind $\rho = \rho_1 + \rho_2(t)$, where $\rho_1$ is constant and $\rho_2(t)$ is small, such that nonlinear terms in (3) can be neglected. With $\omega = v/L$, this gives the differential equation
\[ \frac{1}{\omega^2} \rho_{tt} + 8\rho = 0 \]
which has the solution of a harmonic wave with eigenfrequency $2\sqrt{2}\omega$.

The constant term $\rho_1 = \rho_0 2/3$.

**Proof.** Insertion in (3), and evaluation.

The framework provides a connection between a densification in terms of a soliton ‘particle’, and a harmonic oscillator, (which could be a wave, at a fix spatial point).

**ANALOGY WITH HARMONIC TRAVELING WAVE**

An analogy with a water wave is considered, such that the ratio of angular velocities for Mercury and Venus can be derived.

For a harmonic traveling wave, the frequency is determined by the angular velocity of the particles Falkemo (1980).

Consider Venus and Mercury aligned as in Figure 1, where $u_r$ denote the radial velocity at Mercury and $c_v$ the orbital velocity of Venus.

![Figure 1](image)

From Strömberg (2014), the maximum radial velocity can be derived to read $u_r = f\omega_\text{Mer}(r_{\text{ecc}}/R)$, where $\omega_\text{Mer}$ is the angular velocity of Mercury and $R$ is the mean orbital radius. With eccentricity ratio 1/4.5, and $f=2$, the angular velocity is given...
by \( \omega_m/2.3 \). This is close to \( \omega_m/2.5 \) which coincides with the value for Venus. Hereby, with \( c_v \) being the wave velocity of a traveling wave in horizontal direction, and \( u_r \) being the particle velocity, this is in analogy with a traveling wave, as considered in Falkemo (1980).

A similar analogy between Mars, and the asteroids may also provide relations for eccentricity and the couplings.

**DEFORMATION DUE TO TIDAL EFFECTS**

When moons, comets or meteor satellites are sufficiently close to the primary, the spatial variation of gravity in the object becomes relevant, and a deformation is possible. When the so called tidal force and centrifugal force due to sidereal rotation is of the same magnitude, it is possible that additional dynamical phenomena may occur. The location when these forces are equal is known as the Roche limit Shu (1982). It appears that at this limit, deformations (and internal forces) are large such that the satellite disintegrates into planetary rings, or vanishes. The nature of deformation is highly dependent on the decomposition of the object, e.g. fluid satellites may deform and be more susceptible to loads, whereas solid rocky objects achieve large internal forces, such that explosive conditions may occur, cf.

**Kinematic description of the configuration of water at a tidal wave**

Earth is subjected to tidal deformation with (approximately) half the time period of the sidereal rotation, 12h. With the conditions at Earth as the point of departure, a model for the motion of a fluid element at the surface, will be derived.

*Element at surface of earth moving in a noncircular path.*

From the visualisation of the motion for a fluid element at surface of the earth, c.f. Encyclopedia Britannica, it is seen that it can be described in the same manner as the solution in the 2-step linearisation with the ratio 2 between L-frequency \( \omega_L \), and sidereal rotation \( \omega_0 \), cf. Strömberg (2014). Such a path is given in Figure 2.

![Figure 2: Path with radius vector derived in Strömberg (2014), \( r=r_0+r_e \sin(\omega_L t) \), where \( r_0 \) is the radius of a circle, \( r_e \) is generalized eccentricity and the L-frequency \( \omega_L=2\omega_0 \).](https://www.example.com/figure2.png) Solid line: Ellipsoidal

The concept in Strömberg (2014) provides a plausible model of the configuration of water at a tidal wave. Possibly, it may also be representative for other motions, e.g. solid tides, and dynamics in the atmosphere.

**Link to Roche Limit**

With the definition of Roche limit from Wikipedia, the planet disintegrates since the gravity pull is less than the tidal force of the primary. Consider a tidal oscillation as derived above. If assumed also for more solid objects, this may serve as a refined description, or a pre-state, of the dynamical conditions at the Roche limit, when a moon disintegrates. In modeling, it may be assumed that inertia at a solid tidal deformation is of significant magnitude, such that an oscillation and resonance may occur. When the amplitude is large, this may be interpreted as a rupture/disintegration, and hence, a link to the Roche limit is provided.
Due to difficulties of performing large scale experiments, and in-field observations, details of such states remains to be shown. An example may be the moon Phobos at Mars, which rotates with increased velocity. If this is balanced with a gravity force, a comparison gives an increased gravity pull compared with the classical Newtonian.

CONCLUSION

Models relating wave dynamics to motions in the solar system was deduced.

- For a soliton solution, the density is increased at the traveling wave location, and at formation, a planet may be considered such a densification. A linearization admits harmonic solutions, with an eigenfrequency. Hence, a correspondence between a ‘generalized particle’ and waves, were established.
- The location and orbital motions of Mercury and Venus were connected with a model that relies on the assumptions for a water wave.
- The results from Strömberg (2014) with a secondary frequency for a non-circular orbit were applied in conjunction with tidal effects. It was shown that the water at Earth is described by the path derived in Strömberg (2014), with the L-frequency $\omega_L=2\omega_0$.

REFERENCES


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