Journal of Physics and Astronomy Research
Vol. 1(2), pp. 013-014, October, 2014. © www.premierpublishers.org, **ISSN**: XXXX-XXXX

JPAR

Short Communication

A model for non-circular orbits derived from a two-step linearisation of the Kepler laws

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In the Solar System most orbits are circular, but there are some exceptions. The paper addresses results from a two-step linearisation of the Kepler laws, to model non-circular orbits, at Newtonian gravity and other interactions with adjacent bodies. The orbit will then be characterised by a generalised eccentricity and a secondary frequency denoted L-frequency, ω_L (and considered proportional to the angular velocity). The path will be that of a circle, superimposed by small vibrations with the L-frequency. Hereby, the amplitude corresponds to an eccentricity, such that the radius varies, with time. When the ratio between the L-frequency and angular velocity is a non-integer, 'perihelion' moves. Bounds are derived and resulting orbits are generated and visualized.
For the integer ratio 2, results are compared with an ellipsoidal, and a tidal wave. For a non-integer ratio, the orbit is related to data for Mercury. Methods for detecting and measuring the secondary frequency are discussed, in terms of transfer orbits in Spaceflight dynamics.

Keywords: Kepler laws, planetary orbit, linearization, L-frequency, eccentricity, inertia refinement, perihelion, detection, Mercury, Mars

INTRODUCTION

Newtonian gravity gives elliptic orbits, but the eccentricity is not specified, when no additional conditions. For the 2-body problem, or multi-body problem, other forces act along the path, and resulting equations are complex. A related result is (Bruns-Poincaré´s Theorem, 1887), on impossibility of solving the 3-body problem. Recent developments in nonlinear dynamics indicate that systems can self-organise into steady motions, by means of frequency coupling. Here, it is assumed that a planet satisfies linearized Kepler laws with an additional refinement of inertia part (in some contexts, denoted as perturbation theory).This results in radial oscillations around a circular path. The frequency for oscillation will be denoted, the L-frequency.

Other relative motions with oscillations are e.g. Satellite librations, c.f. Klemperer and Baker (1957), where several dynamical models are given.

EXCEPTIONS FROM CIRCULAR ORBITS

Most of the planets and moons, in Kepler motion have very low eccentricity, such that the orbits are almost circular. Mercury, closest to the sun do not have a circular orbit, and the eccentricity is about 0.2. The planet has a coupling 3:2, for the siderial rotation and orbital rotation. Hereby it is somewhat related to the sun, as a moon to its primary, where coupling is 1:1.
Mars, close to the asteroid belt, has eccentricity approximately 0.1. The other exceptions are two dwarf planets in the asteroid belt, and Pluto.
Next, a linearisation of Kepler laws, which gives noncircular orbits with generalised eccentricity 0.25, and 0.2 will be given.

LINEARISATION OF KEPLER LAWS OF MOTION

In well-known notations, (cf. eg. wikipedia) Kepler law in r-direction reads

$$m(r_{,tt} - (\theta_{,t})^2 r) - F_r = 0 \qquad (1)$$

where m is the mass of orbiting body, r and θ are the radius and angle in the orbit, and F_r is the Newtonian gravity force, $F_r = -mGM/r^2$, G is the universal constant of gravity and M is the central mass.

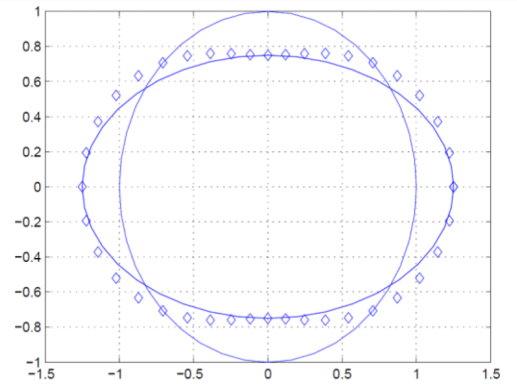

Figure 1. Orbit o, created from radius vector obtained by a two-step linearization with ω_L=2ω.

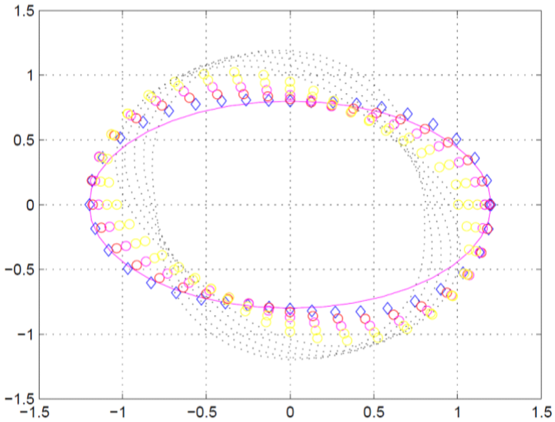

Figure 2. Path from a 2-step linearization with ωL= $(4.2)^{1/2}$W, and generalised eccentricity r_{ecc}=0.2r_0.

Proposition

There are solutions, so called small vibrations r_ε, with a frequency around r = r_0, θ = θ_0, where subscript 0, is the path of a circle satisfying (1). One choice of linearisation gives the frequency 2ω, where ω^2 = GM/r_0^3, is the angular velocity of the orbital rotation, and amplitude r_0/4, and that ω is constant, although the orbit is not circular. This orbit compared with an ellipsoidal is shown in Figure 1. Subsequently, such a frequency will be referred to as the L-frequency, ω_L of a noncircular orbit.

Proof

From Kepler law, the angular momentum is constant, which reads

$$(\theta_{,t})^2 r^4=(\theta_{0,t})^2 r_0^4 \qquad (2)$$

Using (2), expansion of inertia part and *Fr* in (1), and including a nonlinear term from inertia gives

$$r_{\varepsilon,tt} +5\omega^2 r_\varepsilon+ 4\omega^2(r_\varepsilon)^2/r_0= 0 \qquad (3)$$

In the 2nd step of linearisation, r_ε in the nonlinear term is chosen as r_ε=-r_0/4. This gives frequency 2ω and, assuming this location as the innermost point in oscillation, also amplitude.

Remark

In the general case, any frequency and amplitude can be provided by the linearisation, depending on at which point 2nd step of linearisation is done. Orbit could also change, not maintaining a steady motion with one frequency. Such behavior may be exposed by Mercury, as indicated in Correia and Laskar (2011).

RESULTS

Solution to the differential equation for an harmonic oscillator, obtained by a linearization of (3), gives that the expression for an orbit reads

$$r=r_0+r_{ecc}\sin(\omega_L t) \qquad (4)$$

where r_0 is the radius in a circular orbit, r_{ecc} is the generalised eccentricity.

In the figures the orbital paths are shown for two linearisations. In Figure 1, the orbit obtained from a linearisation is shown, compared with ellipsoidals with axis 3r_0/4 and 5r_0/4.

In Figure 2, the linearisation is such that the L-frequency will be $(5-0.8)^{1/2}$ω, and amplitude 0.2r_0. At the beginning, the path is indicated with o, and then small dots.

Neglecting the out-of plane components, the eccentricity agrees with the path for Mercury. It is seen that the location for maximum and minimum radius, rotates, since the frequency is not an integer multiple of ω. Depending on the point of linearisation, when amplitude is $0.25r_0$, the L- frequency will be bounded in an interval such that $2\omega \le \omega_L \le 6^{1/2}\omega$.

METHODS FOR DETECTING THE L-FREQUENCY

Measurments and maneuvering of Earth Satellites.

To change of orbits for satellites, derivations and formulas are given in Wiesel (2010). With additional analysis, and numerical calculations or simplifications, the control variables may be used for detection:

Low Thrust
For low thrust transfer, replace velocity and location with results from two-step, such that dependency on ω_L, ω and t, are obtained.

Modification of the Hohmann transfer.
To enter the transfer ellipsoidal, assume instead that a velocity in radial direction may be applied. The magnitude for this is derived from differentiation of the radius from a two-step linarisation.

Eccentricity for planetary orbits.

More detailed observations and analysis of the paths of Mars and Mercury, e.g. from visibility at Earth on latitude 55, (Modern Astronomi 2014,2, p. 28).

CONCLUSIONS

A two-step generalized linearisation, with adjustment to a specified amplitude was derived. This admits solution into a non-circular path, where radius varies due to small vibrations, with a so-called L-frequency. In conjunction with assuming that nonlinear systems may self-organise, this may serve as a solution to an in-plane multi-body problem in celestial mechanics. The capture into a harmonic solution with one frequency agrees, to some extent, with the KAM-theorem.

- Here, small radial vibrations were considered. In another context (e.g. whirls), it may be of interest to investigate how an original path varies, when subjected to other types of modifications, with a rigorous perturbation theory.
- Methods for detecting the L-frequency, was discussed.
- It is possible that other frequency ratios are related, e.g. sidereal versus orbital rotation for Mercury, Correia and Laskar (2011), and the distance between adjacent planets in terms of ratios of angular velocities. Other recent suggestions for locations of all the planets are models based on differential equations, eg. Schrödinger equation de Oliveira Neto et al., (2004) and modified Laplaces equation, with eigen-values.
- The tide at Earth, may be a manifest of the theory, such that the L-frequency for the water is related to the sidereal rotation of earth by $\omega_L=2\omega$.
- Maintenance of the noncircular motion may require energy supply, which for Mercury, could be related to energy from the Sun, and for other orbits, to additional gravity. This suggests that the capture into circular orbits are preferable when no extra energy is present.
- The fact that most orbits are circular, was maybe, at that time, not entirely understood by Kepler, and therefore the ellipsoidals remain as models for the planetary paths. In the present paper, the Kepler model was used as the point of departure, to derive a new possible model for a noncircular orbit, resulting in equation (4), i.e. small vibrations with the frequency ω_L around a circular orbit with radius r_0 .

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Accepted 29 September, 2014.

Citation: Strömberg L (2014). A model for non-circular orbits derived from a two-step linearisation of the Kepler laws. Journal of Physics and Astronomy Research 1(2): 013-014.